

Practical Design of Post-Tensioned Buildings
12.29.18

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NED – 1983 – CIVIL BATCH

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Seminar Learning Objectives

- ❑ Understand the basic concepts and methodology of Post Tensioning (PT)
- ❑ Learn about the ACI 318 Code guidelines on PT design and construction
- ❑ Learn the procedure for flexural and shear design of post-tensioned beams and slabs
- ❑ To understand the causes of cracking in PT structures and explore methods to mitigate their effects
- ❑ Discuss the practical considerations/ field issues as applied to PT structures

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Course Outline

- Description of PT Systems
- Basic Methodology of Pre-stressing
- Load Balancing Concept
- ACI Code – Permissible stresses and design guidelines
- Post Tensioning Losses
- Secondary Moments
- Flexural Design of PT members
- Shear Design
- Restraint Cracks & their mitigation
 - Pour strips & control joints, Released Joints, Structural Separation
- Barrier Cables with PT Strands
- External Post Tensioning
- Anchorage Zone Design
- PT Slabs on Ground
- Field Issues
 - Prior to Concreting
 - After Concreting

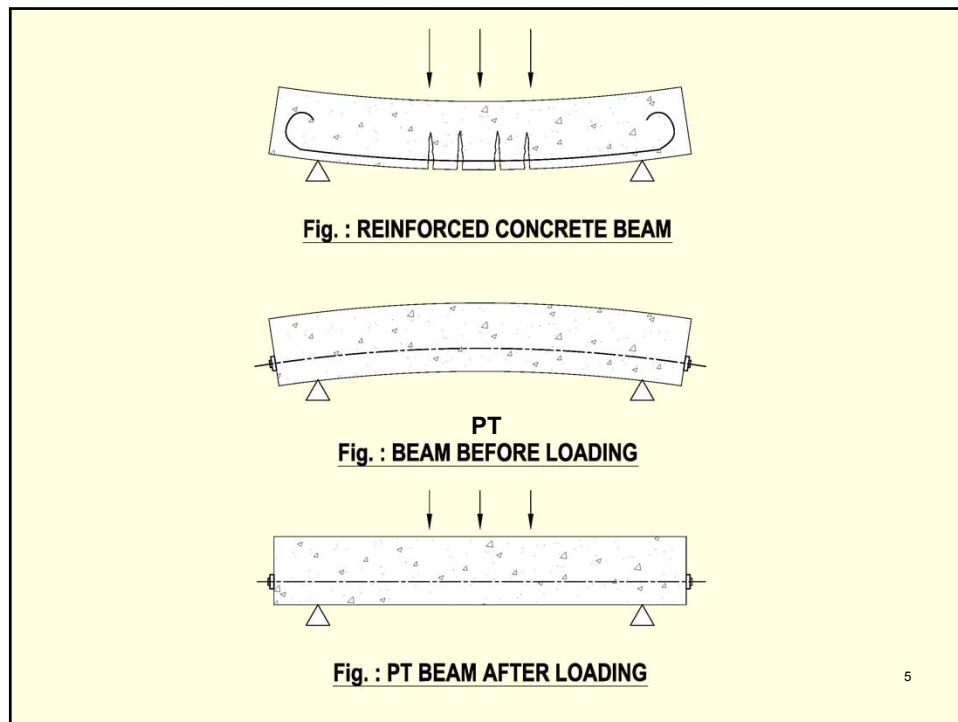
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PRESTRESSING

What is Pre-Stressing?

- Method of reinforcing concrete with high strength steel
- Concrete is strong in compression and weak in tension
- Tensile strength $f_t = 8$ to 10% of f'_c
- Flexural Strength; $f_r = 7.5 \sqrt{f'_c}$ – Modulus of rupture
- Initial compression is introduced in concrete members so that it counters the flexural tension in the extreme fibers resulting from applied loads
- The initial compression is introduced by stretching the high strength steel which induces tension in the steel. The steel imparts an equal and opposite compressive force on concrete.

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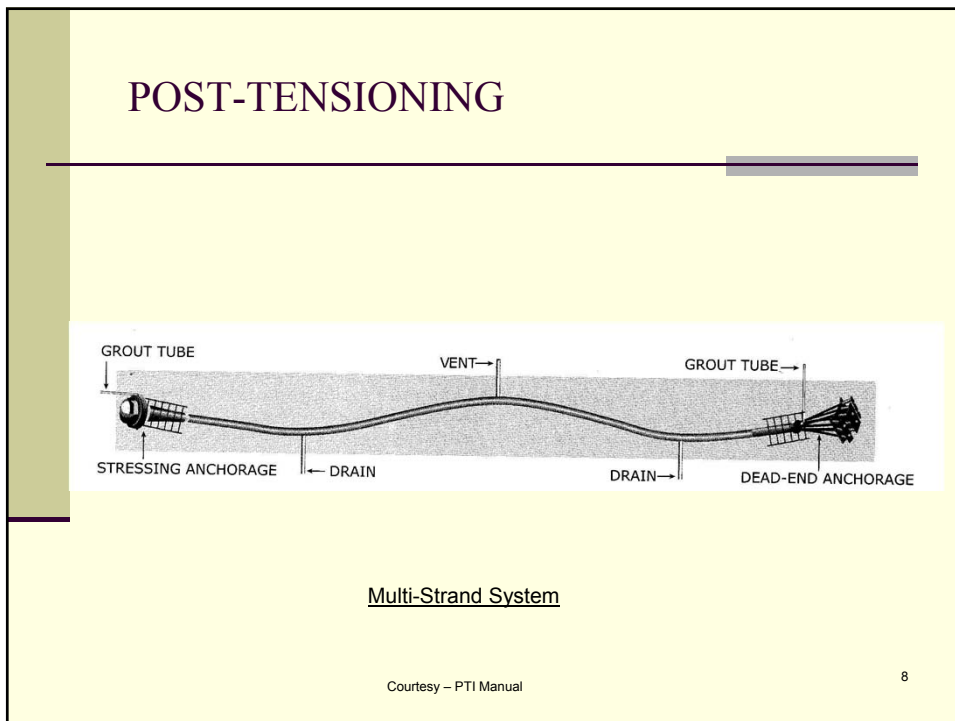
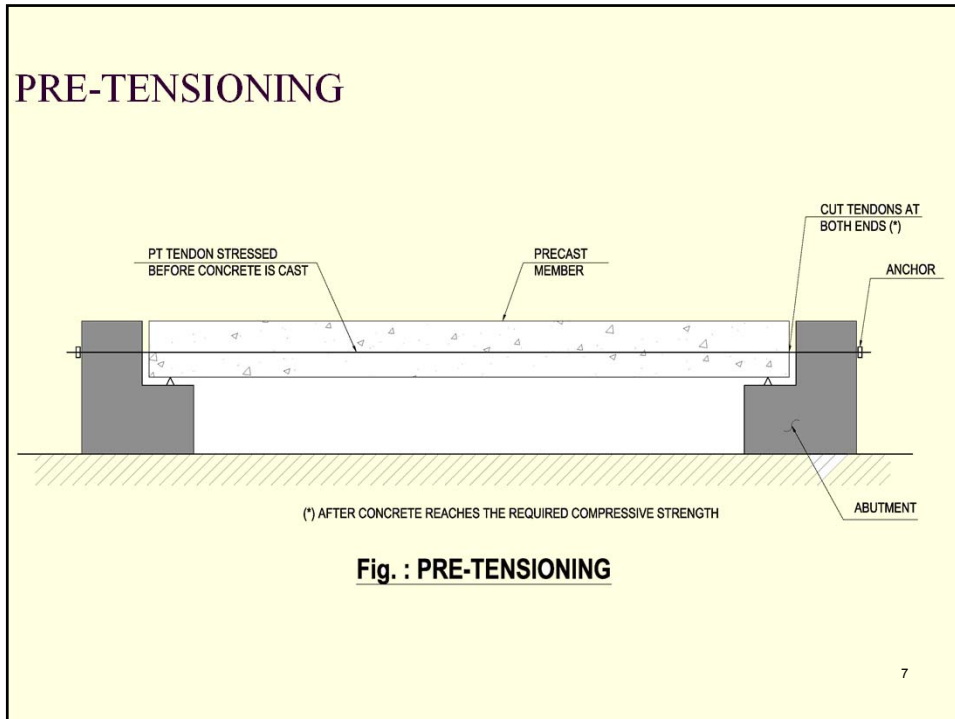
METHODS OF PRESTRESSING

Pre-Tensioning

- Steel is stressed **before concrete is poured**
- It is stretched between abutments and then concrete is cast (see sketch)
- After concrete reaches a specified strength, steel is cut between the ends of the members and the abutments
- This transfers the prestressing force to the concrete.
- This method is usually performed at plants for precast concrete
- **Bonded Tendons**

Post-Tensioning

- Steel is stressed **after concrete is cast and cured.**
- Steel is installed at the job-site inside formwork.
- Steel inside ducts; **Bonded (grouted); Un-bonded (greased) tendons**
- The steel has special anchors (bearing plates) at the ends which attaches it to the concrete
- Steel is stretched to induce forces after the concrete hardens
- This method is usually performed at the site for cast-in-place concrete



POST-TENSIONING SYSTEMS

Types of Post-Tensioning Systems

Internal PT system

- PT tendons are embedded in concrete before concrete is cast
- Bonded or unbonded system – Explained in the next few slides

External PT system

- PT tendons are mounted outside the structural member
- Behavior is similar to unbonded system as the tendon moves independently of concrete
- Anchorages provided at ends of member
- Deviators provided to provide harped profile
- Mostly used in retrofit applications and bridge applications
- Fireproofing is an important consideration in External PT System

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POST-TENSIONING SYSTEMS

Unbonded PT System

- Typically single strand (7 wire) coated with grease and inside ducts
- The duct is an extruded plastic sheathing – the strand free to move inside duct
- Anchored at end – Ductile iron anchor and hardened steel wedges
- Standard or encapsulated tendons
- Encapsulated tendons have a complete protection coating on all the components including end anchorages
- Encapsulated tendons are for aggressive environments and any exposure of tendon to water, chlorides and other harmful substances is avoided.
- Typically used in building applications such as slabs & beams

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POST-TENSIONING SYSTEMS

Bonded PT System

- Typically multiple strands or bars inside ducts
- Larger ducts made of corrugated galvanized steel or HDPE ducts
- Ducts with strands usually placed in member before concrete is poured
- Sometimes ducts only are placed at concrete pour and strands are installed later. More difficult option
- After concrete hardens the tendons are stressed and ducts filled with grout.
- The grout bonds the PT strand to surrounding concrete
- Ports are provided for grout injection
- The grout protects the strands from corrosion
- Typically used in Bridge applications and heavy duty building application such as Transfer Girders
- The bonded tendons furnishes more ultimate moment capacity

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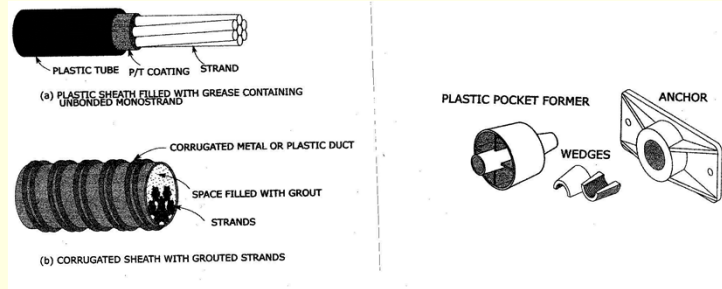
POST-TENSIONING HARDWARE

- Unbonded or bonded tendon
- End Anchorages
- Pocket Former or grommet
- Steel wedges
- Hydraulic Jacks
- Pressure Gages

SKETCHES & PICTURES TO FOLLOW

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POST-TENSIONING SYSTEMS – Strand & End Anchor details

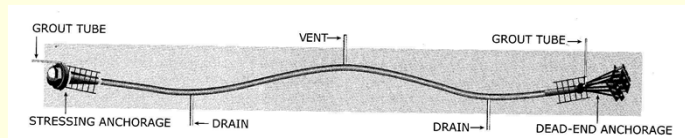


UNBONDED & BONDED CABLES

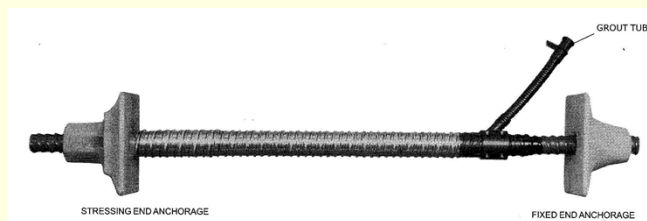
STRESSING END ANCHOR ASSEMBLY

Courtesy – PTI Manual

POST-TENSIONING SYSTEMS – Multi-strand & Bar Tendon



Multi-Strand System Components

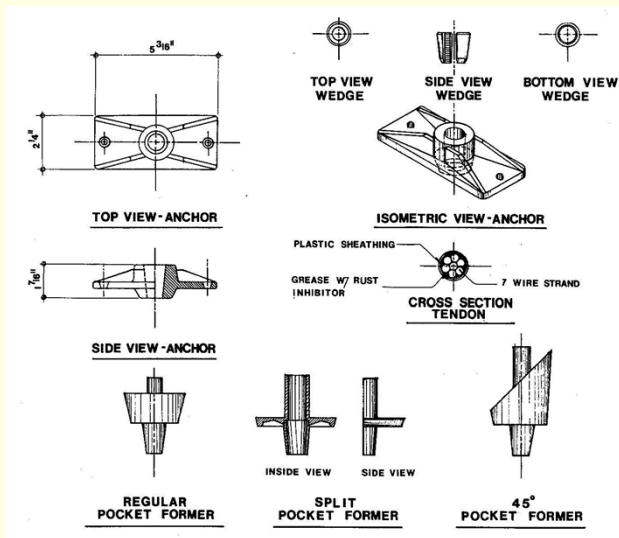


Bar Tendon Assembly

Courtesy – PTI Manual

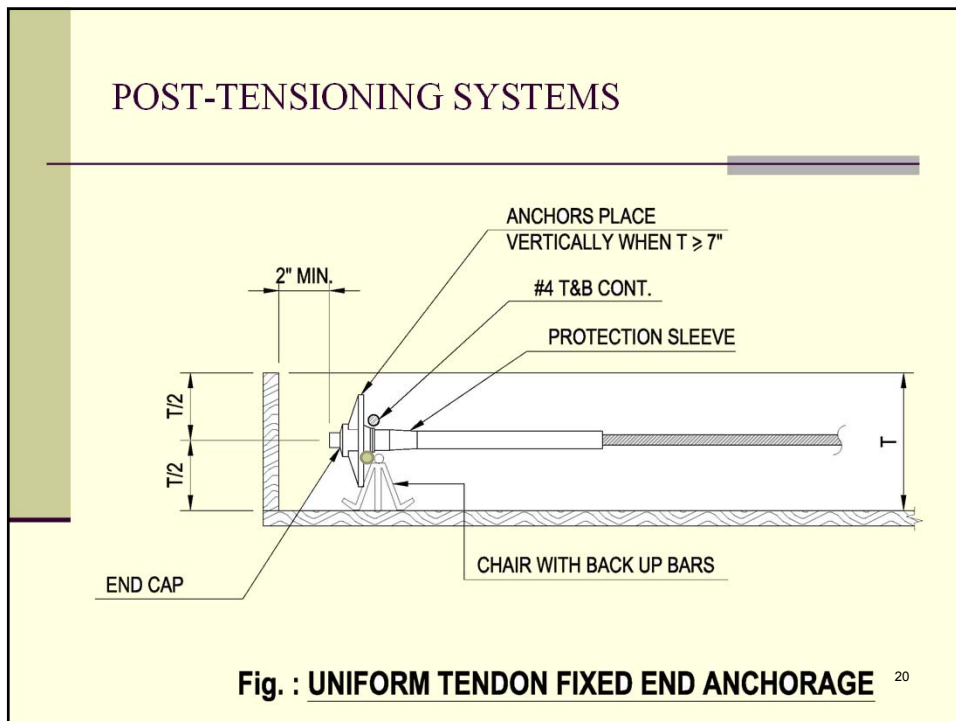
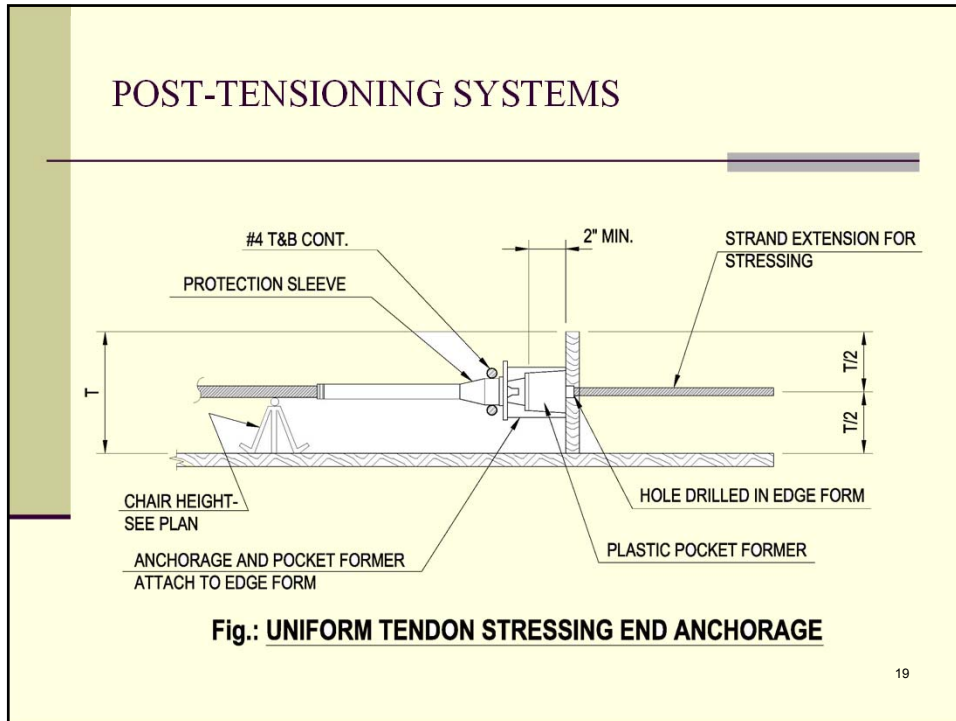


POST-TENSIONING SYSTEMS – Pocket Former & Anchorages



Courtesy – Amsysco





TYPE OF TENDON

Definition of Tendon

Assembly of high strength steel, end anchorage and duct

Types

Single Strand System

- Single strand tendon consists of 7 wires
- Six wires are helically wrapped around a straight wire
- The diameter of strand ranges from 0.375" to 0.6".
- Pocket former – conical device is used at stressing end to form a pocket for stressing with a hydraulic jack. Also called grommet.
- Conical shaped wedges lock the strand into the anchorage after stressing
- The duct is filled with grout if it is a bonded system.

Multiple Strand System

- Multiple strands installed in a single large duct.
- The anchorages for this system are usually specially designed and supplied by the PT supplier
- Large concentrated force in the anchorage zones
- The duct is filled with grout after stressing is done.

Bar Steel

- Single or multiple bars of high strength steel.
- A complete system consists of rods, anchorages and ducts.
- Can be used in bonded and unbonded situations

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BAR TENDON STRESSING EQUIPMENT



Slide courtesy of DSI - America



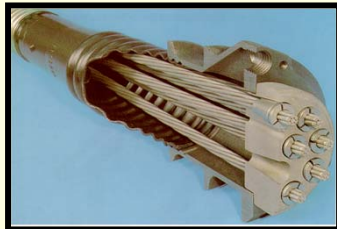
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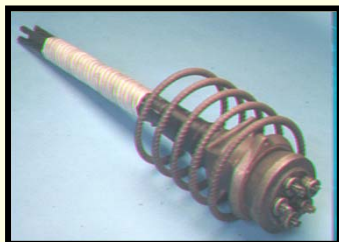
MULTI-STRAND STRESSING



Slide courtesy of DSI - America



Multi Strand Tendons usually Bonded



Bar Tendon



Slide courtesy of DSI - America

PT MATERIALS

I] PRESTRESSING STEEL

a) 7 Wire Strand Properties (ASTM A416)

Ultimate Strength = 250 to 270 ksi

$E = 28.5 \times 10^6$ psi

0.375" to 0.6" dia

Low Relaxation or Stress Relieved Steel

Low Relaxation – typically used in North America – resistant to relaxation

b) Bar Steel

Ultimate Strength = 150 ksi

$E = 28.5$ to 29×10^6 psi

0.625" to 2.5" dia

Used for short straight tendons

Couplers are common

II] CONCRETE

$f'_c = 4000$ psi to 7000 psi

Initial concrete strength at time of stressing = 3000 psi

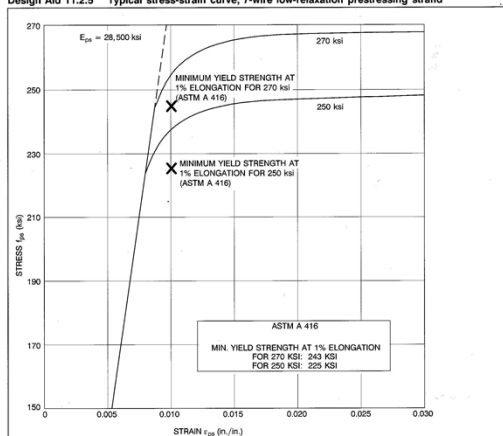
Most typical concrete strength in building applications = 5000 psi

Higher strength causes more volume change (shrinkage/creep) which is detrimental to serviceability

Non-shrink concrete - better

MATERIAL PROPERTIES PRESTRESSING STEEL

Design Aid 11.2.5 Typical stress-strain curve, 7-wire low-relaxation prestressing strand



These curves can be approximated by the following equations:

250 ksi	270 ksi
$\epsilon_{ps} \leq 0.0078 : f_{ps} = 28,500 \epsilon_{ps} \text{ (ksi)}$	$\epsilon_{ps} \leq 0.0086 : f_{ps} = 28,500 \epsilon_{ps} \text{ (ksi)}$
$\epsilon_{ps} > 0.0078 : f_{ps} = 250 - \frac{0.04}{\epsilon_{ps} - 0.0064} \text{ (ksi)}$	$\epsilon_{ps} > 0.0086 : f_{ps} = 270 - \frac{0.04}{\epsilon_{ps} - 0.007} \text{ (ksi)}$

Courtesy – PCI Manual

PRESTRESSING TERMINOLOGY

Partial Prestressing

- PT provided in one direction with mild steel in the other direction
- PT provided less than Code required minimums
 - This is compensated by mild steel
 - The tensile stresses may exceed certain limits

Full Prestressing

- Full PT provided in both directions

Staged Prestressing

- When not all PT tendons are stretched at the same time.
- This is to avoid overstressing initial stresses
- Transfer girders – when the dead loads from all floors are not loading the girder; typ in high rise applications

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PT FRAMING

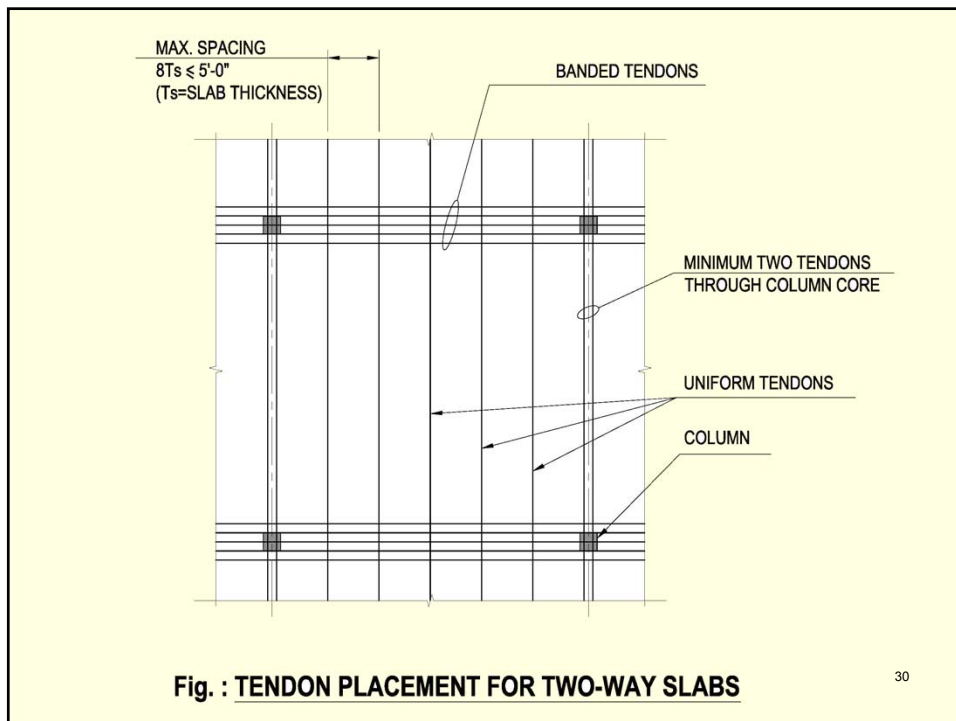
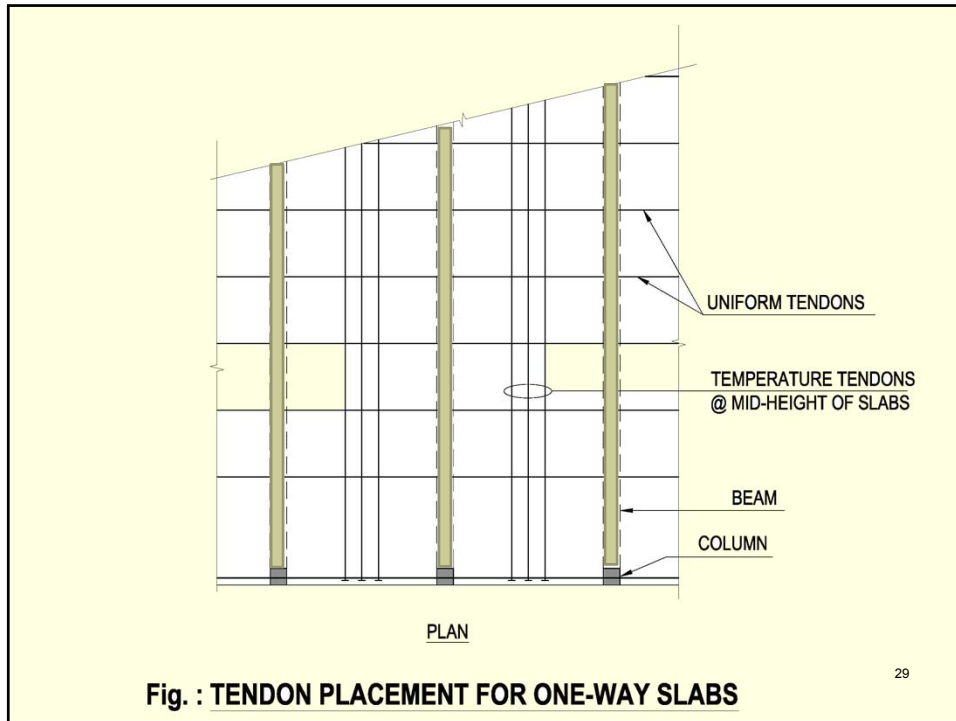
One Way Framing

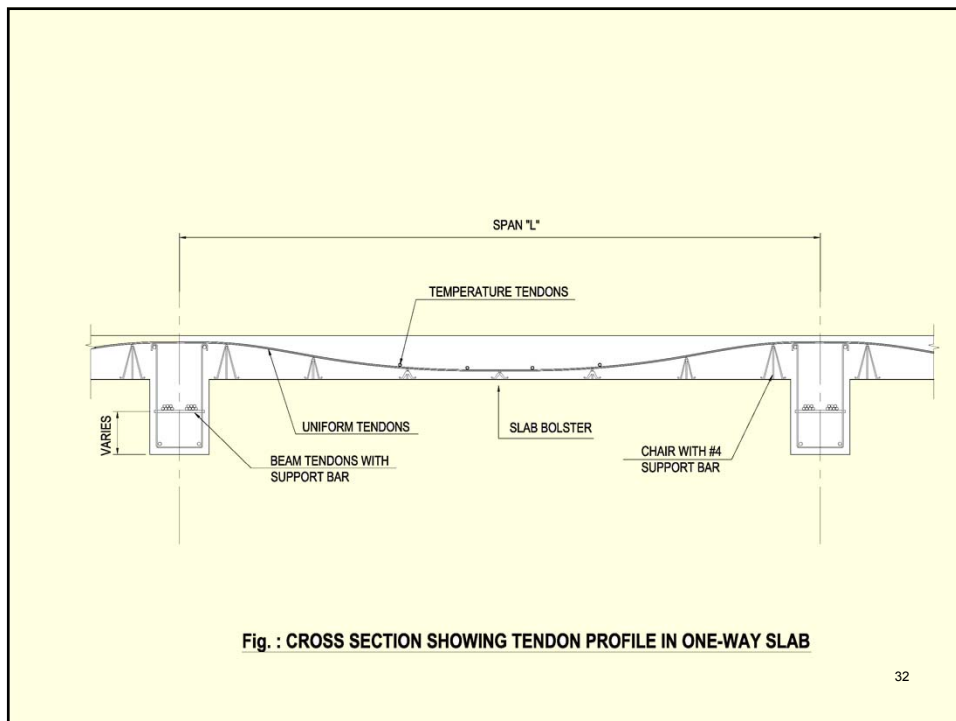
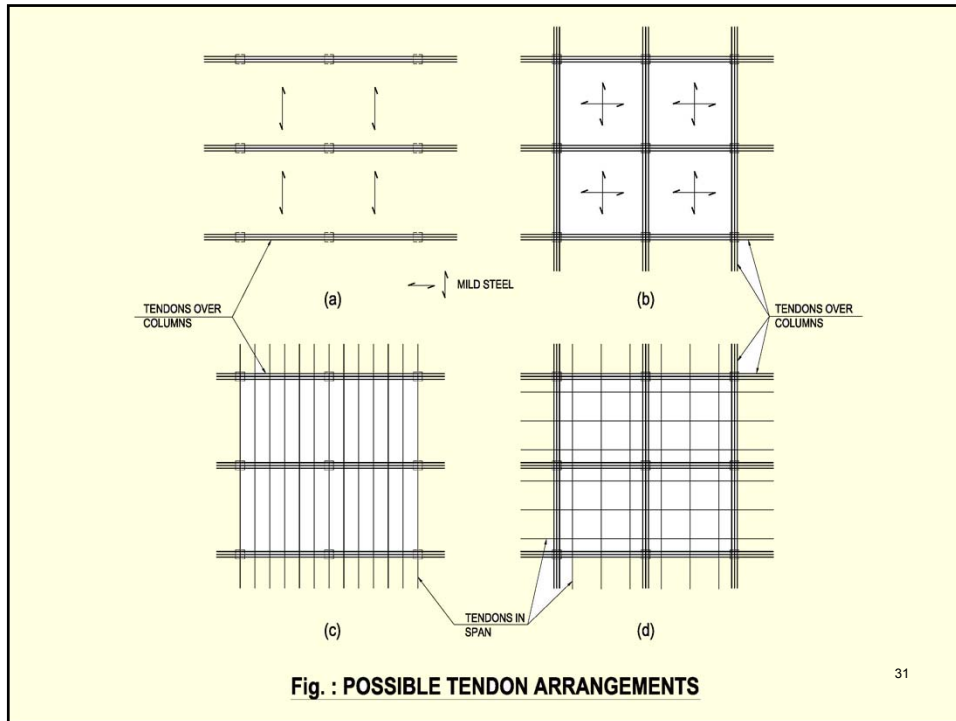
- Slabs on Beams; Beams may be PT or non-PT
- Beams

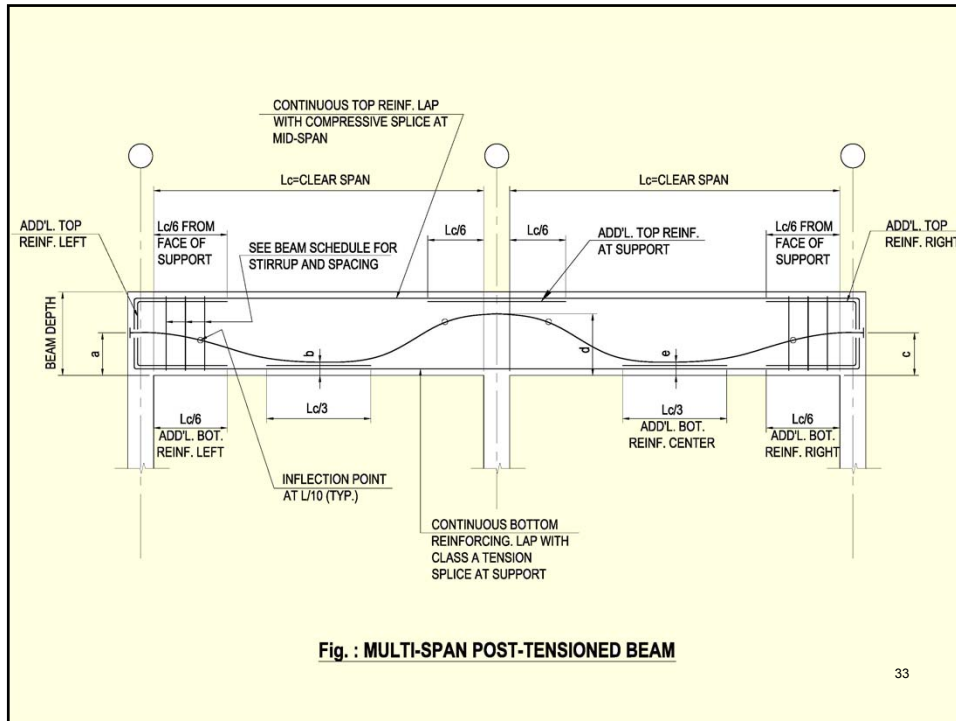
Two Way Framing

- Flat Plates
- Flat Slabs
- Banded or uniform tendon distribution
- More common is banded in one direction and uniform in the other direction

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PT APPLICATIONS











- Buildings including parking structures
- Bridges – Box Girders, Segmental, Cable Stayed, Precast Girders...
- Storage Structures – Water Storage Tanks, Silos, Nuclear containment (Circumferential as well as vertical PT)
- Tension Members – Ring beam and other applications
- Rehab and retrofit applications
- Pavements
- Foundation over expansive & compressible soils
- Rock and Soil Anchors to brace buildings, retaining walls, dams...
- Tie backs for Earth Stabilization
- Masonry Structures – Both vertical and horizontal PT
- Barrier Cables

BREAKDOWN (Approx.)

- 50% structures – 50% slab on grade, foundation and earthwork
- For structures: 2/3 – Buildings and 1/3 – Bridges

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FIELDS OF POST-TENSIONING

Bridges 	Tanks 	Domestic build. 	Commerial build. 	Slope stabilization 
Excavations 	Tunnelling 	Mining 	Hydraulics harbour constr. 	Miscellaneous 

Slide courtesy of DSI - America 35



PROS & CONS OF PRESTRESSING

Pros

- Larger spans between columns compared to reinforced concrete for same slab T
- Thinner members (30% less) resulting in:
 - Lower floor to floor height and overall building height
 - Reduced weight of building – smaller foundations & seismic loads
 - 25-30% reduction in amount of concrete and steel
- Faster floor construction cycle – Formwork can be stripped as soon as the member is stressed & concrete reaches 75% of specified 28 day strength
- Less shear reinforcement because of the uplift provided by tendons
- Less congestion in laying reinforcement
- Lesser number of cracks because of pre-compression

Cons

- Special skill set required for laying out and stressing PT
- Special equipment to build
- Have to be careful with openings
- Future large openings tricky
- Materials may be more expensive
- Need good Anchorage for tendons; have to contend with bursting stresses

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CONCRETE FIBER STRESS DISTRIBUTION Rectangular Beam with Straight Tendon

Fig. (a)
Concentric tendon – prestress only

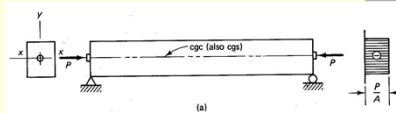


Fig. (b)
Concentric tendon with gravity load

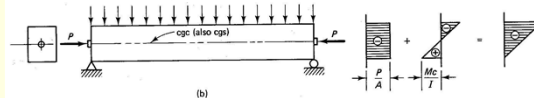


Fig. (c)
Eccentric tendon – prestress only

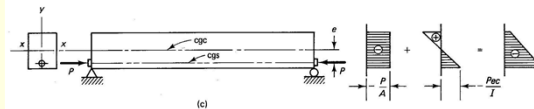
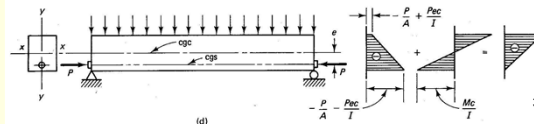


Fig. (d)
Eccentric tendon with gravity load



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BASIC METHODOLOGY OF PRESTRESSING

Compressive forces are introduced into the member to counteract the tension stresses resulting from the dead weight of the member and the externally applied loads.

Consider a simply supported rectangular beam subjected to a concentric prestressing force P. This results in a uniform compressive stress on the beam cross-section which is given by:

$$f = - P/A \quad \rightarrow \text{Eq. 1}$$

where:

A = bh is the cross-sectional area of a beam section of width b and total depth h.

If external loads are applied on the beam causing a maximum moment M at mid-span, the resulting stresses are:

$$f_t = - P/A - M/S_t = - P/A - M*c/I_g \quad \rightarrow \text{Eq. 2}$$

$$f_b = - P/A + M/S_b = - P/A + M*c/I_g \quad \rightarrow \text{Eq. 3}$$

where: f_t = stress at top fibers
 f_b = stress at bottom fibers
 c = distance from centroidal axis to extreme fiber = h/2 for rectangular section
 I_g = gross moment of inertia of section = $bh^3/12$ for rectangular section

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BASIC METHODOLOGY OF PRESTRESSING

As we know, concrete is good in compression but weak in tension. So Eq. 2 which shows that the top fibers of the cross-section are in compression is not controlling the design.

However, the Eq. 3 which introduces tension in the bottom fibers of the section due to the externally applied loads plus the beam self weight, controls the design.

After a certain load value the beam bottom fibers will exceed the allowable values for tension. In order to avoid this limitation and extend the capacity of the beam, the prestressing tendon is placed eccentrically below the neutral axis at midspan. This has the effect of inducing tensile stresses at the top fibers and compression stresses at the bottom fibers due to prestressing. See figure which illustrates this point.

If the tendon is placed at eccentricity "e" from the center of gravity of the concrete (cgc), the moment created because of the offset tendon is $P*e$ and the resulting stresses at midspan using the principle of superposition of stresses are:

$$f_t = - P/A - M*c/I_g + P*e*c/I_g \quad \rightarrow \text{Eq. 4}$$

$$f_b = - P/A + M*c/I_g - P*e*c/I_g \quad \rightarrow \text{Eq. 5}$$

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BASIC METHODOLOGY OF PRESTRESSING

Since the support section of a simply supported beam has zero moment (and as such zero stresses in the extreme fibers) from the externally applied load, the eccentric prestressing force induces a large tensile force in the top fibers at this section.

To counter this effect, the eccentricity of the prestressing tendon profile (cgs line) is reduced or even reversed at the support section. This is done gradually in the form of a smooth parabolic line which is at its low point at midspans and then lifted up at the support lines at each end. This process of shaping the tendon profile is called draping.

Sometime for concentrated loads the tendon abruptly changes its profile to provide an upward force to counter the concentrated load and this process is called harping of the tendon.

The above equations 4 & 5 can be modified for initial conditions and for service load conditions. Assume at initial condition the prestressing force is P_1 (no losses) while the moment on the section is the dead load moment, M_d . The equations are:

$$f_t = - P_1/A - M_d*c/I_g + P_1*e*c/I_g \quad \rightarrow \text{Eq. 6}$$

$$f_b = - P_1/A + M_d*c/I_g - P_1*e*c/I_g \quad \rightarrow \text{Eq. 7}$$

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BASIC METHODOLOGY OF PRESTRESSING

For service load conditions, the tendons have undergone some losses and its force is the effective force P_e while the moment on the section at this stage include the superimposed dead load moment, M_{sd} and the live load moment, M_L in addition to the dead load moment, M_d

Thus,

$$M_T = M_d + M_{sd} + M_L$$

$$f_t = - P_e/A - M_T*c/I_g + P_e*e*c/I_g \quad \rightarrow \text{Eq. 8}$$

$$f_b = - P_e/A + M_T*c/I_g - P_e*e*c/I_g \quad \rightarrow \text{Eq. 9}$$

The use of these equations are illustrated with an example on the next page

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BASIC METHODOLOGY OF PRESTRESSING

Example 1 – Computation of Extreme Fiber Stresses in a Prestressed Beam by the Basic Method

Design Data

Rectangular Beam Properties

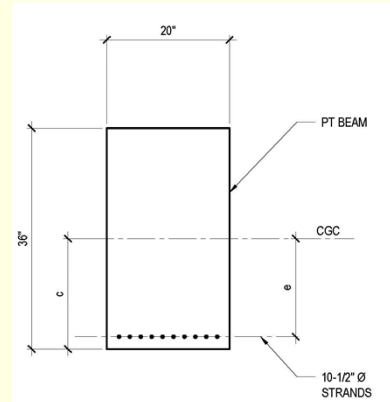
$b = 20''$ & $h = 36''$
 $A = 720 \text{ in}^2$
 $c = 18''$
 $e = 15''$
 $I_g = 77,760 \text{ in}^4$

Span & Loading data

$L = 40'$
 $w_d = 0.75 \text{ k/ft}$ [self weight]
 $w_{sd} = 0.50 \text{ k/ft}$ [super imposed load]
 $w_L = 2.00 \text{ k/ft}$ [live load]
 $w_T = 3.25 \text{ k/ft}$ [total load]

Material Properties

$f'_c = 4 \text{ ksi}$
 $f_{pu} = 270 \text{ ksi}$ [low lax strand]
 $f_{pi} = 189 \text{ ksi}$ [Initial stress = $0.70 * f_{pu}$]
 $f_{pe} = 159 \text{ ksi}$ [Service stress assuming 30 ksi in losses; losses on high side]
 Assume 10 - $\frac{1}{2}''$ dia 7 wire strands; $A_{ps} = 10 * 0.153 = 1.53 \text{ in}^2$



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BASIC METHODOLOGY OF PRESTRESSING

INITIAL CONDITION – ASSUMING NO LOSSES

Note that there will be some losses for initial condition (ES, SL & Friction but they have been ignored here)

$$M_d = w_d * L^2 / 8 = 0.75 * 40^2 / 8 = 150 \text{ ft-kips}$$

$$P_i = A_{ps} * f_{pi} = 1.53 * 189 = 289.17 \text{ kips}$$

From Equation 6 & 7

$$f_t = - P_i / A - M_d * c / I_g + P_i * e * c / I_g = - 289.17 / 720 - (150 * 12 * 18 / 77760) + (289.17 * 15 * 18 / 77760)$$

$$= - 0.402 - 0.417 + 1.005$$

$$= +0.185 \text{ ksi (Tension at top fibers)}$$

$$> 3 \sqrt{f'_c} = 0.164 \text{ ksi assuming } f'_{ci} = 3000 \text{ psi}$$

$$f_b = - P_i / A + M_d * c / I_g - P_i * e * c / I_g = - 289.17 / 720 + (150 * 12 * 18 / 77760) - (289.17 * 15 * 18 / 77760)$$

$$= - 0.402 + 0.417 - 1.005$$

$$= -0.990 \text{ ksi (Compression at bottom fibers) } < 0.60 f'_{ci}$$

OK

FINAL SERVICE LOAD CONDITION – AFTER PRESTRESS LOSSES

$$M_T = w_T * L^2 / 8 = 3.25 * 40^2 / 8 = 650.00 \text{ ft-kips}$$

$$P_e = A_{ps} * f_{pe} = 1.53 * 159 = 243.27 \text{ kips}$$

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BASIC METHODOLOGY OF PRESTRESSING

From Equation 8 & 9

$$f_t = - P_e/A - M_T*c/I_g + P_e*e*c/I_g = -243.27/720 - (650 * 12 * 18 / 77760) + (243.27 * 15 * 18 / 77760)$$

$$= -0.338 - 1.805 + 0.844$$

$$= -1.299 \text{ ksi (Compression at top fibers)}$$

< $0.6 \sqrt{f_c}$ assuming $f_c = 4000 \text{ psi}$

$$f_b = - P_e/A + M_T*c/I_g - P_e*e*c/I_g = -243.27/720 + (650 * 12 * 18 / 77760) - (243.27 * 15 * 18 / 77760)$$

$$= -0.338 + 1.805 - 0.844$$

$$= +0.623 \text{ ksi (Tension at bottom fibers)}$$

$$= 9.8 \sqrt{f_c} > 7.5 \sqrt{f_c} \text{ but } < 12 \sqrt{f_c}$$

= Class T

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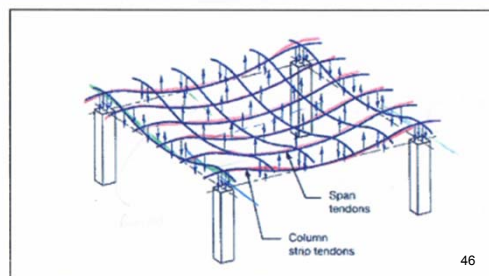
LOAD BALANCING CONCEPT

- Introduced by T.Y. Lin
- Basic Concept – The tendon's influence on structure can be represented by a series of equivalent loads.
- The equivalent loads are axial compression and uplift loads that balance a part of the load on the structure.
- In effect, the load balanced structure is viewed as a non-prestressed member with a reduced loading and an axial compression

Powerful concept

Simplifies calculations

Applicable to draped or harped tendons



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LOAD BALANCING CONCEPT

- If 100% of load is balanced then there will be no bending or deflection. The stresses at any point of the beam will be P_e/A where P is the applied prestressing force and A is the cross-sectional area of the member.
- If only a portion of the member load is balanced then the stress at any point along the member will be

$$f_t = - P_e/A - M_{ub} * c/I_g \rightarrow \text{Eq. 10}$$

$$f_b = - P_e/A + M_{ub} * c/I_g \rightarrow \text{Eq. 11}$$

where: M_{ub} is the moment caused by the net load. This moment is called the unbalanced load moment.

- It can be shown that for a tendon with a parabolic profile, the upward push provided by the tendon is:

$$w_b = 8 * P_e * e / L^2$$

[Solution of a differential equation]

LOAD BALANCING CONCEPT

If the applied load (including member self weight) matches this load then the beam will be in pure compression. However, if the imposed load exceeds the uplift (balancing load) then the unbalanced load is given as:

$$w_{ub} = w_T - w_b \text{ [Net Load]}$$

$$M_{ub} = w_{ub} * L^2/8$$

The stresses in the top and bottom fibers of the beam can be calculated by Eq. 10 & 11 respectively.

In reality a high percentage of the dead load is balanced by the Load Balancing Concept and then the stresses are checked for the unbalanced load. This percentage usually ranges from 70% - 100% of the dead load with 90% being the more common value

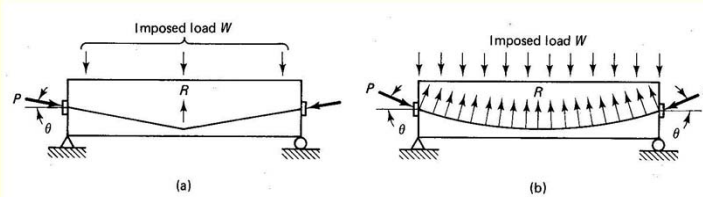


Fig.- Load Balancing Forces (a) Harped Tendons (b) Draped Tendons

LOAD BALANCING CONCEPT

Example 2 – Using the same beam as in Example 1, compute the Extreme Fiber Stresses by the Load Balancing Method

Design Data

Rectangular Beam Properties

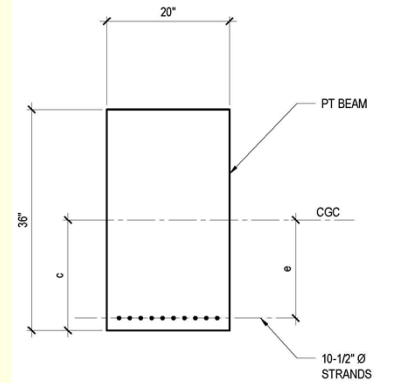
b = 20"
 h = 36"
 A = 720 in²
 c = 18"
 e = 15"
 I_g = 77760 in⁴

Span & Loading data

L = 40'
 w_T = 3.25 klf [total load]

Stresses

f_{pu} = 270 ksi [low lax strand]
 f_{pi} = 189 ksi [Initial stress = 0.70* f_{pu}]
 f_{pe} = 159 ksi [Service stress assuming 30 ksi in losses]
 Assume 10 – 1/2" dia 7 wire strands; A_{ps} = 10* 0.153 = 1.53 sq. in



LOAD BALANCING CONCEPT

From Example 1:

P_e = 243.27 kips
 e = 15"
 c = 18"

Balanced Load: w_b = 8* P_e * e/L² = [8 * 243.27 * 15/12]/ 40² = 1.52 k/ft

If the applied load on the beam (including self weight) were 1.52 k/ft, then the beam would not have any bending or deflection and will be in pure compression.

$$w_{ub} = w_T - w_b = 3.25 - 1.52 = 1.73 \text{ k/ft}$$

$$M_{ub} = w_{ub} * L^2/8 = 1.73 * 40^2 / 8 = 345.9 \text{ ft-kips}$$

From Eq. 10 & 11

$$f_t = - P_e/A - M_{ub}*c/I_g = - 243.27 / 720 - 346*12*18 / 77760$$

$$= - 0.338 - 0.96 = -1.299 \text{ ksi (compression)}$$

this is the same value as before (see example 1)

$$f_b = - P_e/A + M_{ub}*c/I_g = - 243.27 / 720 + 346*12*18 / 77760$$

$$= - 0.338 + 0.96 = + 0.623 \text{ ksi (tension)}$$

this is the same value as before (see example 1)

MUCH SIMPLER CALCS

DESIGN – INVESTIGATION STAGES

PT MEMBERS	REINFORCED CONCRETE MEMBERS
<p>Three major stages</p> <ul style="list-style-type: none"> ■ Jacking/ Transfer stage ■ Service Load stage ■ Factored Load stage 	<p>Two major stages</p> <ul style="list-style-type: none"> ■ Service Load stage ■ Factored Load stage

For investigation of stresses at transfer of prestress, at service loads and at cracking loads, elastic theory shall be used with the following assumptions:

- Strains vary linearly with depth through the entire load range
- At cracked sections, concrete resists no tension

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SERVICEABILITY REQUIREMENTS

As per ACI 18.3.3

Prestressed flexural members shall be classified as Class U or T or C based on the computed extreme fiber stress in tension (f_t) in the pre-compressed tensile zone calculated at service loads as follows:

- Class U – Uncracked $\rightarrow f_t \leq 7.5\sqrt{f_c}$
- Class T – Transition $\rightarrow 7.5\sqrt{f_c} < f_t \leq 12\sqrt{f_c}$
- Class C – Cracked $\rightarrow f_t > 12\sqrt{f_c}$

Prestressed two-way slab systems shall be designed as Class U with $f_t \leq 6.0\sqrt{f_c}$.

For Class U & Class T members, stresses at service loads shall be permitted to be calculated using uncracked section.

For Class C members, stressed at service loads shall be calculated using cracked transformed section

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SERVICEABILITY REQUIREMENTS

Both concrete and prestressing tendon stresses are limited to ensure satisfactory behavior immediately after transfer of prestress and at service loads

As per ACI 18.4.1

AT TRANSFER

Stresses in concrete immediately after prestress transfer and before time-dependent prestress losses:

- Extreme fiber stress in compression $\leq 0.6 f'_{ci}$
- Extreme fiber stress in compression at ends of simply supported members $\leq 0.7 f'_{ci}$
- Where computed tensile strength, $f_t > 6\sqrt{f'_{ci}}$ at ends of simply supported members or $3\sqrt{f'_{ci}}$ at other locations additional bonded reinforcement shall be provided in the tensile zone to resist the total tensile force in concrete.

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SERVICEABILITY REQUIREMENTS

As per ACI 18.4.2

AT SERVICE LOADS

Compression in Concrete

Stresses in concrete for Class U and Class T members at Service Loads based on uncracked section properties and after allowance of all prestress losses shall not exceed:

- Extreme fiber stress in compression due to prestress plus sustained load
 $f_c \leq 0.45 f_c$
- Extreme fiber stress in compression due to prestress plus total load
 $f_c \leq 0.60 f_c$

Tension in Concrete

Tensile stress as per ACI 18.3.3

- Class C - The spacing requirements of bonded reinforcement for prestressed members with calculated tensile stress exceeding $12\sqrt{f_c}$ shall not exceed that given in section 10.6.4

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PERMISSIBLE STRESSES IN PRESTRESSING STEEL

As per ACI 18.5

Tensile stress in prestressing steel (A416) shall not exceed the following:

- Due to prestressing steel jacking force = $0.94 f_{py}$
but not greater than the lesser of $0.80 f_{pu}$ and the max value from the mfr.
- Post-tensioning tendons at anchorage devices
and couplers immediately after force transfer = $0.7 f_{pu}$

Note:

For low relaxation wire and strands: $f_{py} = 0.9 f_{pu}$

For stress-relieved wires, strands and plain & deformed bars the above limits are lower and is often taken as: $f_{py} = 0.85 f_{pu}$ (subject to manufacturer)

SERVICEABILITY DESIGN REQUIREMENTS ACI 318-11 TABLE - R18.3.3

	Prestressed			Nonprestressed
	Class U	Class T	Class C	
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 18.3.4	Gross section 18.3.4	Cracked section 18.3.4	No requirement
Allowable stress at transfer	18.4.1	18.4.1	18.4.1	No requirement
Allowable compressive stress based on uncracked section properties	18.4.2	18.4.2	No requirement	No requirement
Tensile stress at service loads 18.3.3	$\leq 7.5\sqrt{f'_c}$	$7.5\sqrt{f'_c} < f_t \leq 12\sqrt{f'_c}$	No requirement	No requirement
Deflection calculation basis	9.5.4.1 Gross section	9.5.4.2 Cracked section, bilinear	9.5.4.2 Cracked section, bilinear	9.5.2, 9.5.3 Effective moment of inertia
Crack control	No requirement	No requirement	10.6.4 Modified by 18.4.4.1	10.6.4
Computation of Δf_{ps} or f_s for crack control	—	—	Cracked section analysis	$M/(A_s \times \text{lever arm})$, or $0.6f_y$
Side skin reinforcement	No requirement	No requirement	10.6.7	10.6.7

SUGGESTED SPAN TO DEPTH RATIOS – PTI MANUAL

Table 9.3 - Suggested Span/Depth Ratios*

Floor System	Span/Depth Ratio	For L = 30'
One-way slabs	48	→ 7 1/2"
Two-way slabs	45	→ 8"
Two-way slab with drop panel (minimum drop panel at least L/6 each way)	50	→ 7 1/4"
Two-way slab with two-way beams	55	→ 6 1/2"
Two-way waffle slab (5 ft × 5 ft grid)	35	→ 10 1/4"
Narrow & deep beam → Beams, $b \approx h/3$	20	→ 18"
Shallow & wide beam → Beams, $b \approx 3h$	30	→ 12"
One-way joists	40	→ 9"

*These values apply for members with LL/DL ratios < 1.0

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POST-TENSIONING LOSSES

- Prestressing tendons are initially stressed to a percentage of their ultimate strength. Losses in stress occurs immediately after stressing and continues through the life of the structure.
- Cursory treatment of PT losses in ACI 318-11

Types of Losses

Short Term

- Losses which occur between the time the stressing jack engages the tendon and the time the initial prestress force is transferred to the concrete by the anchorage.
- Includes: Elastic Shortening of Concrete; Seating Loss at wedges & Friction Loss

Long Term

- Losses which occur beyond the short term losses
- Includes: Concrete volume change caused by shrinkage & creep and relaxation of PT steel

Important to account for Losses:

- These losses can significantly affect the behavior of a member at service loads.

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POST-TENSIONING LOSSES - Short Term

ELASTIC SHORTENING OF CONCRETE (ES)

For Post-Tensioned Members with unbonded tendons

$$ES = K_{es} * E_s * f_{cpa} / E_{ci}$$

$K_{es} = 0.5$ when tendons are tensioned in sequential order to same tension. For other procedures K_{es} may range from 0 to 0.5

E_s = Modulus of Elasticity of Prestressing steel; 28.5×10^6 psi

f_{cpa} = Average compressive stress in the concrete along member length at c.g. of prestressing steel right after jacking

E_{ci} = Modulus of Elasticity of Concrete at time prestress is applied = $57000 * \sqrt{f'_{ci}}$

ANCHORAGE SLIP - SEATING LOSS (SL)

When PT force is applied to the tendons the wedges slip before they are locked in at the anchors

SL loss is specific to Post-Tensioned Members

PT Manufacturer should provide the anticipated slip, Δ_a .

Usually ranges from 1/4" to 3/8"

$$SL = \Delta_a * E_s / L$$

L = Length of the member

Note that since L is in the denominator, the Seating Loss is significant for short tendons

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POST-TENSIONING LOSSES - Short Term

FRICITION LOSS (FR)

FR loss is specific to Post-Tensioned Members and is caused by friction between the tendon and the duct

The force along a cable is calculated as:

$$P_x = P_s * e^{-(\mu\alpha + k l_x)} \rightarrow \text{ACI 318-08 Eq 18-1}$$

If $(\mu\alpha + k l_x)$ is not greater than **0.30** then the following simplified formula may be used

$$P_x = P_s / (1 + \mu\alpha + k l_x) \rightarrow \text{ACI 318-08 Eq 18-2}$$

Friction Loss between the jacking end and point x along the length of the member

$$FR = P_s - P_x = P_s * (1 - e^{-(\mu\alpha + k l_x)})$$

P_s = Prestressing force at jacking end

P_x = Prestressing force at point x along the length of the member

μ = Curvature friction co-efficient \rightarrow see R18.6.2; ACI 318-08

α = total angular change in radians from jacking end to point x

k = wobble friction coefficient per ft of tendon \rightarrow see Table R18.6.2; ACI 318-08

= **0.001 to 0.002** k/ft. for unbonded tendons

l_x = length of cable from jacking end to point x in feet

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POST-TENSIONING LOSSES - Short Term

FRICTION LOSS (FR)

Important points to consider

- ❑ Friction and wobble coefficients vary depending on (a) type of duct (b) tendon fabrication (c) installation & (d) stressing procedures.
- ❑ PT supplier normally responsible for doing friction loss calculations but the designer needs to know the losses for tendon runs which are larger than normal.
- ❑ For tendons up to **120'** long and stressed from one end or for Tendons up to **220'** long and stressed from both ends, may use average value of prestress for design.
- ❑ If tendon larger than **120'** are stressed from one end or tendons larger than **220'** but stressed from both end, then the friction losses get to be significant and calculated values should be used to arrive at the prestress force in the tendon.

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POST-TENSIONING LOSSES - Long Term

CREEP OF CONCRETE (CR)

For Post-Tensioned Members with unbonded tendons

$$CR = K_{cr} * E_s / E_c * f_{cpa}$$

$K_{cr} = 1.6$ – if light weight concrete then value of K_{cr} is reduced by 20%

E_c = Modulus of Elasticity of Concrete at 28 days

SHRINKAGE OF CONCRETE (SH)

For Post-Tensioned Members

$$SH = 8.2 \times 10^{-6} * K_{sh} * E_s [1 - 0.06 * V/S] * (100 - RH)$$

K_{sh} = Shrinkage Coefficient – see table (day count starts from end of moist curing to application of P/S)

V/S = Volume to Surface ratio, normally taken as gross cross-sectional area of concrete member divided by its perimeter

RH = Average relative humidity ambient to the concrete member = about 73 % for Chicago area

Days	3	5	7	10	30	60
K_{sh}	0.85	0.8	0.77	0.73	0.58	0.45

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POST-TENSIONING LOSSES - Long Term

RELAXATION OF TENDONS (RE)

The relaxation of steel depends on Stress Level and Type of Steel & is interdependent on other losses as well

$$RE = [K_{re} - J(SH+CR+ES)] * C$$

K_{re} & J = coefficients dependent on type of steel; from table below

SH = stress loss due to shrinkage of concrete

CR = stress loss due to creep of concrete

ES = stress loss due to elastic shortening of concrete

C = coefficient dependent on stress level;

for Low Relaxation strand; $f_{pi}/f_{pu} = 0.8 \rightarrow C=1.28$ and $f_{pi}/f_{pu} = 0.74 \rightarrow C = 0.95$

Note the larger losses for Stressed Relieved Strands

Tendon Type	K_{re} (psi)	J
Low Relax strand – 270 grade	5000	0.04
Low Relax wire – 250 grade	4630	0.037
Stress-Relieved strand or wire – 270 grade	20000	0.15
Stress-Relieved strand or wire – 250 grade	18500	0.14

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POST-TENSIONING LOSSES

Short Term Losses = ES + SL + FR

Long Term Losses = CR + SH + RE

Total Losses = (ES + SL + FR) + (CR + SH + RE)

Lump Sump Losses

- ACI starting with the 83 Code is not recommending the use of lump sump losses.
- AASHTO and PTI - Ok to use lump sump losses
- Roughly: Pre-tensioning – 30 ksi, Post-tensioning – 20 ksi (low-relax)

[Post-tensioning losses are smaller because some shrinkage has already taken place before PT is applied and because of use of low-relax tendons

Table 3.1—Approximate prestress loss values (PTI 1990)

Post-tensioning material	Prestress loss, psi	
	Slabs	Beams and joists
Stress-relieved 270k strand and stress-relieved 240k wire	30,000	35,000
Bar	20,000	25,000
Low-relaxation 270k strand	15,000	20,000

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POST-TENSIONING LOSSES

The values in the Table given on the previous page do not include:

- Friction Loss (FR)
- Anchor Seating Loss (SL)

Note: Overestimation of Prestress losses can be almost as detrimental as under-estimation because the former can result in excessive camber.

Example 3:

Calculating Average Prestress Force Furnished by a 0.5" dia strand after losses

Recall that from ACI 18.5

Tensile stress in prestressing steel (A416) shall not exceed the following:

- Immediately after prestress transfer = $0.70 f_{pu}$

$$f = 0.70 * 270 = 189 \text{ ksi}$$

Assuming 15 ksi in total losses

$$f_e = 189 - 15 = 174 \text{ ksi}$$

$$\text{Force provided by the 0.5" dia. tendon} = A_{ps} * f_e = 0.153 * 174 = 26.6 \text{ kips}$$

As per ACI 18.6.2.1 the required effective prestress force shall be indicated in the Contract Documents.

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One & Two Ended Stressing

The 27.0 kips average value for a 0.5" dia. tendon is used when the PT tendons are stressed as follows:

- $L < 120'$ → One end stressing
- $120' < L < 220'$ → Two End stressing
- $L > 220'$ → Use Intermediate stressing points with two end stressing or use lower value of the stress in the prestressing steel based on the actual losses. Friction losses will be substantial for longer lengths.

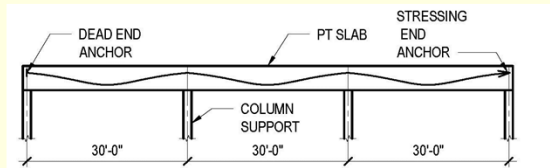


FIG.: ONE END STRESSING - $L \leq 120'$

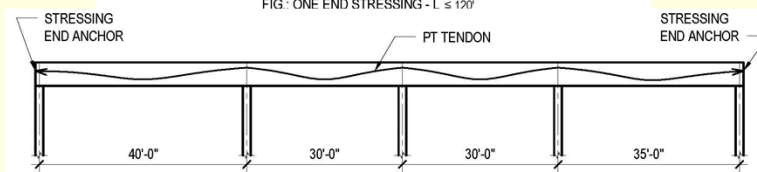


FIG.: TWO END STRESSING - $120' < L < 220'$

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MINIMUM BONDED REINFORCEMENT

As per ACI 18.9

A minimum area of bonded reinforcement shall be provided in all flexural members with unbonded tendons. This is to ensure flexural performance at ultimate member strength and to limit crack width and spacing at service load.

For beams and one way slab systems (ACI 18.9.2)

Minimum area of bonded reinforcement:

$$A_s = 0.004 A_{ct}$$

Where:

A_{ct} is the area of that part of cross-section between the flexural tension face and c.g of gross section

This bonded reinforcement shall be uniformly distributed in the tensile zone as close as possible to the extreme tension fiber. The reinforcement is required in the tension zone of both positive and negative moment regions

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MINIMUM BONDED REINFORCEMENT

For two way slab systems – positive moment areas (ACI 18.9.3.2)

- Bonded reinforcement is not required in positive moment areas where the extreme fiber stress in tension at service loads does not exceed $2 \sqrt{f'_c}$
- When the extreme fiber stress in tension at service loads in positive moment areas exceeds $2 \sqrt{f'_c}$ the minimum area of bonded steel is computed as:

$$A_s = N_c / (0.5 f_y)$$

where: N_c is the tension force in concrete due to unfactored dead load plus live load

For two way slab systems – negative moment areas (ACI 18.9.3.3)

- The minimum area of bonded reinforcement in top of slab in each direction shall be computed as:

$$A_s = 0.00075 A_{cf}$$

where: A_{cf} is the larger gross cross-sectional area of slab-beam strips in two orthogonal equivalent frames intersecting at a column

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MINIMUM BONDED REINFORCEMENT

Bonded reinforcement in the negative moment area shall satisfy the following:

- They shall be distributed between the lines that are $1.5h$ (h =slab thickness) outside opposite faces of the column support.
- A minimum of 4 bars shall be provided in each direction
- Spacing of reinforcement shall not exceed 12"

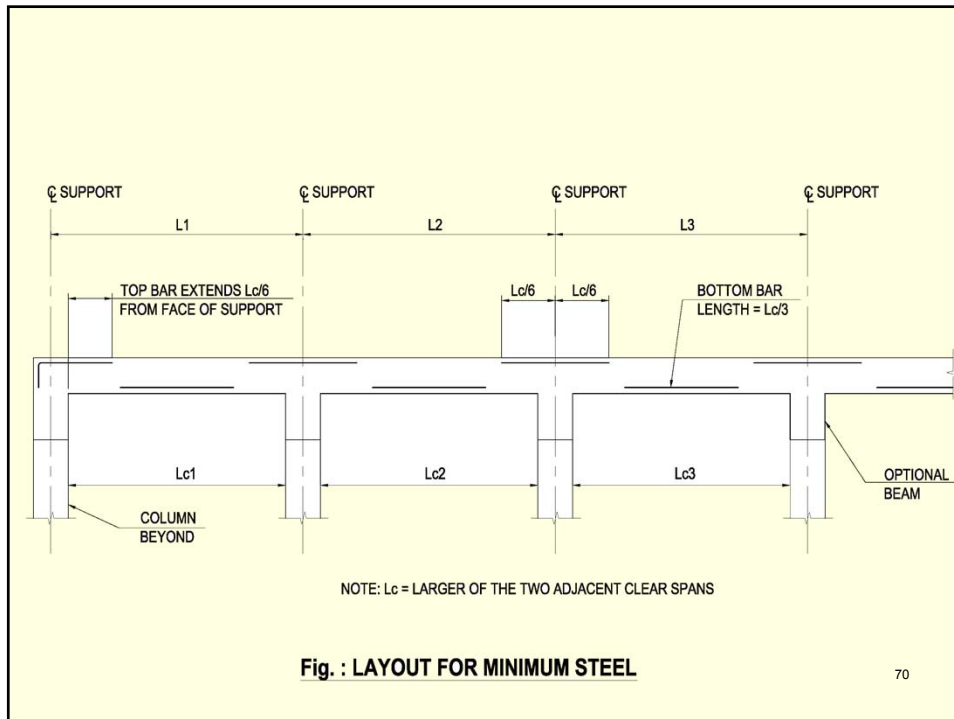
Minimum length of bonded reinforcement (ACI 18.9.4)

In positive moment areas minimum length of bonded reinforcement
 = $1/3^{\text{rd}}$ of clear span length L_n and centered at peak moment

In negative moment areas minimum length of bonded reinforcement
 = $1/6^{\text{th}}$ of clear span L_n on each side of column or wall support

Where bonded reinforcement is provided for factored moment requirement or when the tensile force in mid-span exceeds $2\sqrt{f'_c}$ then the minimum lengths shall also satisfy the development length requirements of Chapter 12

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FLEXURAL STRENGTH DESIGN OF PRESTRESSED MEMBERS

As per the ACI Code the design moment strength of prestressed flexural members may be computed using strength equations similar to those for non-prestressed concrete members. For prestressing steel, f_{ps} shall be substituted for f_y in strength computations.

Since prestressing steel does not have a well defined yield point, stress in the prestressed reinforcement at nominal strength (corresponding a max concrete strain of 0.003) will vary depending on the amount of prestressing. This value of f_{ps} can be obtained using conditions of equilibrium and strain compatibility approach which is quite complex especially for unbonded tendons.

As an alternative, the ACI Code allows f_{ps} to be obtained by approximate equations as described below:

For members with bonded tendons

$$f_{ps} = f_{pu} [1 - \gamma_p \{ \rho_p * f_{pu} / f'_c + d/d_p(\omega - \omega') \} / \beta_1] \quad (18-1)$$

For illustration purposes assume that there is no mild steel reinforcement in the flexural section ($\omega = \omega' = \rho f_y / f'_c = 0$) the above equation reduces to:

$$f_{ps} = f_{pu} [1 - \gamma_p * \rho_p * f_{pu} / f'_c / \beta_1]$$

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FLEXURAL STRENGTH DESIGN OF PRESTRESSED MEMBERS

where

- γ_p = factor for type of prestressing steel
- = 0.55 for high strength prestressing bars ($f_{py} / f_{pu} \geq 0.80$) ASTM A722
- = 0.40 for stress-relieved wire and strands and plain bars ($f_{py} / f_{pu} \geq 0.85$) ASTM A416
- = 0.28 for low-relaxation wire and strands ($f_{py} / f_{pu} \geq 0.90$) ASTM A416

and β_1 is the concrete stress block parameter varying with f'_c

- β_1 = 0.85 for $f'_c \leq 4000$ psi
- β_1 = 0.80 for $f'_c \leq 5000$ psi
- β_1 = 0.75 for $f'_c \leq 6000$ psi
- ω = $\rho \times f_y / f'_c$ → tension reinforcement index
- ω' = $\rho' \times f_y / f'_c$ → compression reinforcement index
- ρ_p = $A_{ps} / b \times d_p$ → prestressing steel reinforcement ratio
- f_{pu} = ultimate strength of prestressing steel
- d_p = Effective depth of prestressing steel $\geq 0.8 * h$
- h = depth of member

For members with unbonded tendons

The equation for stress in prestressing steel is:

$$f_{ps} = f_{se} + 10,000 + f'_c / (100 \rho_p) \quad (18.2) \quad \rightarrow L/d \leq 35 \quad [\text{Mostly Beams}]$$

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FLEXURAL STRENGTH DESIGN OF PRESTRESSED MEMBERS

$$f_{ps} = f_{se} + 10,000 + f'_c / (300 \rho_p) \quad (18.3) \quad \rightarrow L/d > 35 \quad [\text{Mostly Slabs}]$$

f_{ps} shall not be greater than the lesser of:

- f_{py} and $(f_{se} + 60000)$ for Eq (18.2)
- f_{py} and $(f_{se} + 30000)$ for Eq (18.3)

where:

f_{se} is the effective stress in prestressing steel after allowance of all prestress losses
 Note that Eq (18.3) applies to members with high span to depth ratios ($L/d > 35$) such as post-tensioned one way slabs, flat plates and flat slabs while Eq (18-2) typically applies to beams

Once the value of f_{ps} is known the nominal moment strength of a rectangular section or a T-section where the stress block is within the compression flange can be calculated as follows:

$$M_n = A_{ps} * f_{ps} [d_p - a/2]$$

where a = the depth of the equivalent rectangular stress block = $A_{ps} * f_{ps} / (0.85 * b * f'_c)$

Note the above equations for nominal moment strength is similar to that for reinforced concrete with the exception that the yield strength for steel is replaced by the stress in the prestressing steel f_{ps} at nominal moment.

FLEXURAL STRENGTH DESIGN OF PRESTRESSED MEMBERS

If non-prestressed reinforcement is used in the section then the moment strength equation becomes:

$$M_n = A_{ps} * f_{ps} [d_p - a/2] + A_s * f_y [d - a/2] + A'_s * f_y [a/2 - d']$$

$$a = [(A_{ps} * f_{ps}) + (A_s * f_y) - (A'_s * f_y)] / (0.85 * b * f'_c)$$

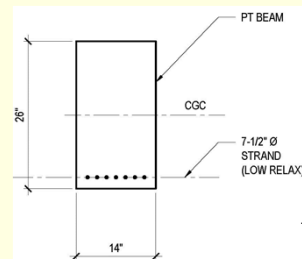
The above equation is true for rectangular beams or for the condition when the stress block is within the flange thickness of the beam for flanged beams.

Example 4: – Computation of Flexural Strength of Prestressed Member shown in sketch assuming both bonded and unbonded tendons:

Design Data

- $b = 14''$
- $h = 26''$
- $d_p = 23''$
- $f'_c = 5000 \text{ psi}$
- $f_{pu} = 270,000 \text{ psi (low relax)}$
- $f_{py} = 0.9 f_{pu}$
- $L = 30'$

NO MILD STEEL REINF.



FLEXURAL STRENGTH DESIGN OF PRESTRESSED MEMBERS

BONDED TENDONS

The ACI Eq 18.1 reduces to the following equation if non-prestressed reinforcement is not considered or is not present:

$$f_{ps} = f_{pu} [1 - \gamma_p * \rho_p * f_{pu} / (f'_c * \beta_1)]$$

where:

$$\begin{aligned} \gamma_p &= 0.28 \text{ for low relaxation strand} \\ \beta_1 &= 0.80 \text{ for } f'_c \leq 5000 \text{ psi} \\ \rho_p &= \text{prestressing steel reinforcement ratio} \\ &= A_{ps}/bd_p \\ &= 7 * 0.153 / (14 * 23) \\ &= 0.00333 \end{aligned}$$

Thus

$$f_{ps} = 270 [1 - 0.28 * 0.00333 * 270 / (5 * 0.8)] = 253 \text{ ksi}$$

Now calculating the nominal moment strength from the equations given above:

$$a = A_{ps} * f_{ps} / (0.85 * b * f'_c) = 7 * 0.153 * 253 / (0.85 * 14 * 5) = 5.3''$$

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FLEXURAL STRENGTH DESIGN OF PRESTRESSED MEMBERS

$$\begin{aligned} M_n &= A_{ps} * f_{ps} [d_p - a/2] \\ &= 7 * 0.153 * 253 [23 - 5.3/2] \\ &= 5514 \text{ in-kips} \\ &= 460 \text{ ft-kips} \end{aligned}$$

Prestressed concrete sections are classified as either tension-controlled, transition or compression-controlled sections in accordance with ACI 10.3.3 and 10.3.4. The strength reduction factor ϕ is based on this classification.

Sections are tension-controlled if the net tensile strain in the extreme tension steel is equal to or greater than 0.005 when the concrete in compression reaches its assumed strain limit of 0.003. The ϕ value of 0.9 is used for tension controlled sections

Sections are compression controlled if the strain in steel is 0.002 or less ($\phi = 0.65$ typ except $\phi = 0.75$ for sections with spiral reinforcement). The ϕ values can be interpolated between the two limits

Tension controlled sections are preferred because compression controlled sections have less ductility and are more sensitive to variations in concrete strength. As such lower values of ϕ are assigned to compression controlled sections

For the section in this example

$$\begin{aligned} c/d_p &= (a/\beta_1)/d_p = 5.3/0.8/23 = 0.288 < 0.375 \\ \text{Tension controlled section; } \phi &= 0.9 \text{ [ACI 9.3.2.1]} \end{aligned}$$

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FLEXURAL STRENGTH DESIGN OF PRESTRESSED MEMBERS

$$\phi M_n = 0.9 * 460 = 414 \text{ ft-kips}$$

UN-BONDED TENDONS

$$L/D = 30/2.17 = 13.82 \leq 35 \rightarrow \text{Eq. 18.2 applies}$$

$$f_{ps} = f_{se} + 10,000 + f'_c / (100 \rho_p) \text{ Eq. (18.2)}$$

Assume the losses have been calculated as 32 ksi

Thus

$$f_{se} = (0.8 * f_{pu}) - 32 = (0.8 * 270) - 32 = 184 \text{ ksi} = 184,000 \text{ psi}$$

$$\rho_p = 0.00333 \text{ same as above}$$

$$f_{ps} = f_{se} + 10,000 + f'_c / (100 \rho_p)$$

$$= 184,000 + 10,000 + 5000/100/0.00333$$

$$= 209,000 \text{ psi} = 209 \text{ ksi}$$

f_{ps} shall not be greater than the lesser of: f_{py} and $(f_{se} + 60000)$

Again, the nominal moment strength is calculated as:

$$a = A_{ps} * f_{ps} / (0.85 * b * f'_c) = 7 * 0.153 * 209 / (0.85 * 14 * 5) = 3.76''$$

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FLEXURAL STRENGTH DESIGN OF PRESTRESSED MEMBERS

$$M_n = A_{ps} * f_{ps} [d_p - a/2]$$

$$= 7 * 0.153 * 209 [23 - 3.76/2]$$

$$= 4727 \text{ in-kips}$$

$$= 394 \text{ ft-kips}$$

$$\phi M_n = 0.9 * 394 = 375 \text{ ft-kips}$$

Note that the unbonded tendons give less ultimate moment capacity compared to bonded tendons

Note for sections reinforced with Bonded reinforcement: [ACI 18.8.2]

The total amount of prestressed and non-prestressed reinforcement shall be adequate to develop a factored load at least 1.2 times the cracking load computed based on the modulus of rupture value of $f_r = 7.5 \sqrt{f'_c}$

This provision is a precaution against abrupt flexural failure developing immediately after cracking
The cracking moment M_{cr} is computed as:

$$M_{cr} = [f_r + F_{se}/A_c] S_b + F_{se} * e$$

- F_{se} = Effective prestress force
- A_c = Area of concrete section
- S_b = Section modulus at bottom of member
- e = eccentricity of prestress force

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FLEXURAL STRENGTH DESIGN OF PRESTRESSED MEMBERS

The $1.2M_{cr}$ requirement for flexural members is waived for members with shear and flexural strength at least twice of that required;

Flexural Strength: $\phi M_n \geq 2 M_u = 2 (1.2M_d + 1.6M_L)$

Shear Strength: $\phi V_n \geq 2 V_u = 2 (1.2V_d + 1.6V_L)$

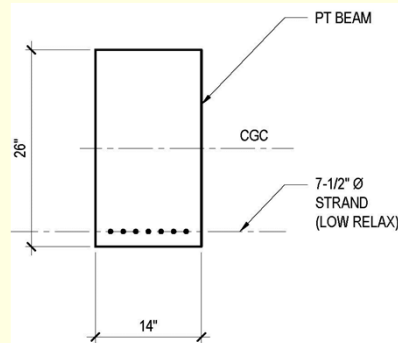
The above requirement that the total reinforcement be adequate to develop a factored load at least equal to $1.2M_{cr}$ does not apply to members with unbonded tendons. Tests of one way slabs and beams have shown that unbonded tendons do not rupture or yield at the time of first flexural cracking.

FLEXURAL STRENGTH DESIGN OF PRESTRESSED MEMBERS

Example 5: – Computation of Cracking Moment Strength and checking of Minimum Reinforcement Limit for PT Member with Bonded Reinforcement:

Design Data – Same beam as in Example 4

- $b = 14''$
- $h = 26''$
- $d_p = 23'' > 0.8h$
- $f'_c = 5000 \text{ psi}$
- $f_{pu} = 270,000 \text{ psi (low relax)}$
- $f_{py} = 0.9 f_{pu}$
- $L = 30'$
- Assume 15% total losses



$$M_{cr} = (f_r + F_{se}/A_c) \cdot S_b + (F_{se} \cdot e)$$

$$f_r = 7.5 \sqrt{f'_c} = 530 \text{ psi}$$

$$F_{se} = .85 \cdot [A_{ps} \cdot 0.7 f_{pu}] = .85 \cdot [7 \cdot .153 \cdot (0.7 \cdot 270)] = 172 \text{ kips}$$

$$S_b = b \cdot h^2 / 6 = 14 \cdot 26^2 / 6 = 1577 \text{ in}^3$$

$$A_c = b \cdot h = 14 \cdot 26 = 364 \text{ in}^2$$

**FLEXURAL STRENGTH DESIGN OF PRESTRESSED MEMBERS
– Contd.**

$$e = 13 - 3 = 10''$$

$$M_{cr} = (f_r + F_{se}/A_c) * S_b + (F_{se} * e)$$

$$M_{cr} = (0.53 + 172/364) * 1577 + (172 * 10) = 1581 + 1720 = 3301 \text{ in-k} = 275 \text{ ft-k}$$

Computation of the cracking moment strength is necessary to check the minimum reinforcement requirement per ACI 18.8.2

ACI 18.8.2 – The total reinforcement (PT & Mild steel) must be adequate to develop a design moment strength at least equal to 1.2 times the cracking moment strength.

From Example 4

$$\begin{aligned} \text{The design moment strength of PT member} &= \phi M_n = 0.9 * 460 = 414 \text{ ft-kips} \\ &> 1.2 * M_{cr} \\ &> 1.2 * 275 = 330 \text{ ft-kips} \\ &\text{OK} \end{aligned}$$

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THE CONCEPT OF SECONDARY MOMENTS

- ❑ In Simple beams no support reactions are induced by prestressing as only the internal stresses are affected by PT.
- ❑ However, in continuous, statically indeterminate beams the prestressing induces a support reaction. This induced reaction produce moments in the beam which are known as Secondary moments.
- ❑ The moment in the beam due to the eccentricity of the prestress is called the Primary Moment. This moment exists in both simple and continuous beams.
- ❑ The prestress on a beam can be replaced by forces acting on the beam and when these forces are analyzed we get the resulting moments. Thus,

Resulting Moment = Primary Moment + Secondary Moments

$$M = M_1 + M_2$$

where:

$$M_1 = F e_1$$

e_1 = Eccentricity of PT force from c.g. of concrete section

Thus, Secondary Moments

$$M_2 = M - M_1$$

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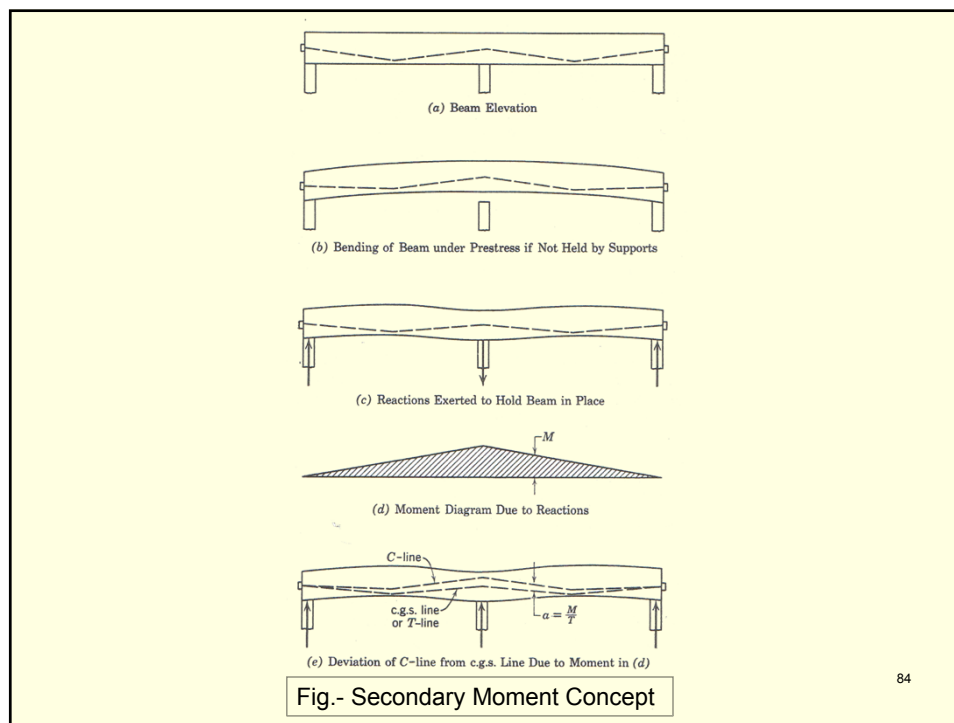
SECONDARY MOMENTS

- ❑ As an illustration of the Secondary Moment concept consider the figure shown on next page which shows a 2 span beam which tends to lift up when prestressed. Reactions are exerted by the support to hold the beam down. These reactions create the "secondary moments" in the beam.
- ❑ Since the secondary moments are created by reactions at the support, they are straight line diagrams.
- ❑ In a simple beam, the line of pressure in the concrete (C-line) coincides with the T-line (c.g. of steel). In a continuous beam the C-line deviates from the T-line by a distance

$$a = M/T \text{ (see sketch for explanation)}$$

- ❑ Note that the term "Secondary moment" is misleading as it may lead one to believe that the moments are secondary in magnitude or un-important but that is not correct, as the secondary moments can be significant

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SECONDARY MOMENTS

- ❑ The computation of moments due to prestress in a continuous beam is explained in figure on the next page. First the primary moment is plotted with the known PT force. The shear diagram resulting from this is next sketched as shown in (c). The resulting loading on the beam is shown in (d). This loading then causes the moment diagram illustrated in (e).
- ❑ Once the resulting moment diagram is known, the secondary moment can be worked out by the equation: $M_2 = M - M_1$
- ❑ Secondary Moments tend to increase the negative moments at the support and decrease the positive moments in the span

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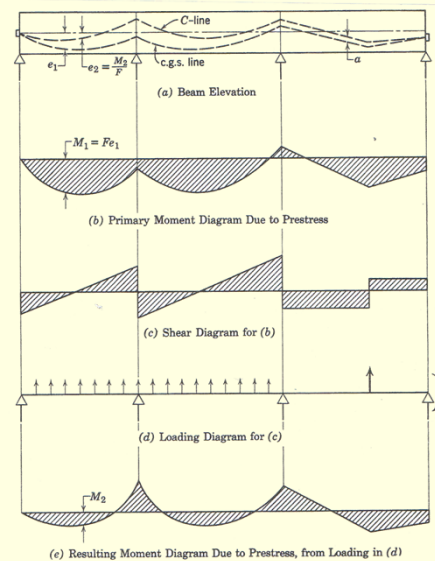


Fig. – Resulting Moment Diagram

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CASE STUDY - HIGH RISE BUILDING

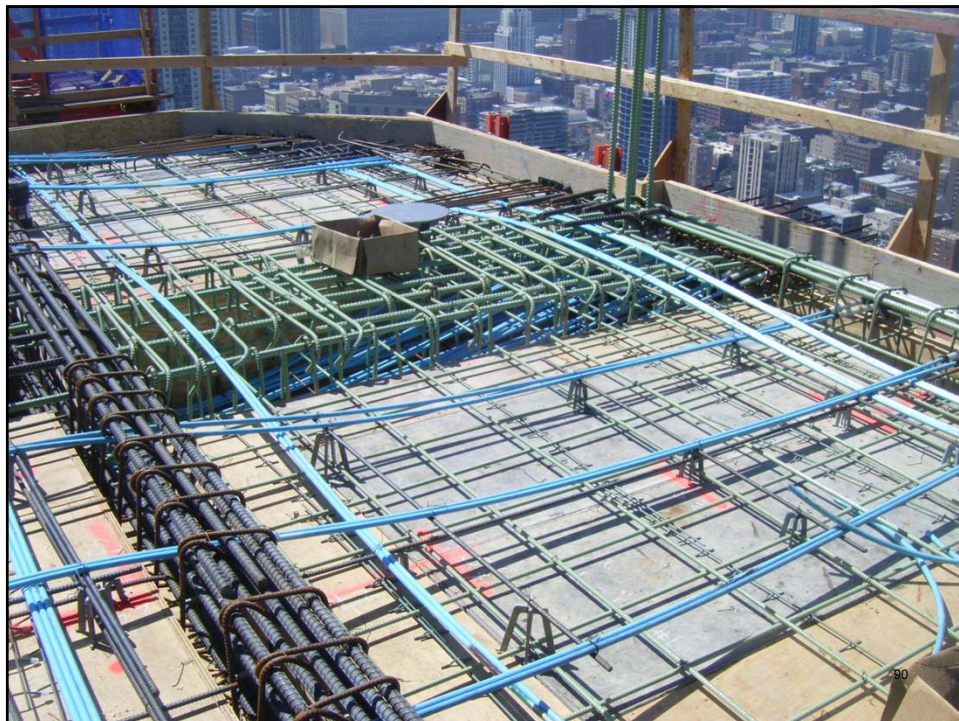
- 66 Story high rise building in Chicago
- Hotel, Condominium & Commercial
- Tuned Liquid Damper atop building
- PT tower
 - To control deflections
 - Thinner flat plate
 - Economical
 - Long spans

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HIGH RISE BUILDING



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DESIGN OF TWO WAY PT SLAB SYSTEM

Example 6 : Design pre-stressed flat plate using the Equivalent Frame method:

Design Data

$f'_c = 5000$ psi
 $f_{pu} = 270,000$ psi (low relax, 7 wire strand)
 $f_y = 60,000$ psi

Residential Floor Slab
 Live Load = 40 psf
 Partition Load = 15 psf
 Misc Load = 5 psf

Spans and Columns as shown in figure
 Concrete Cover = 0.75" typical u.n.o
 Concrete Cover End Spans = 1.5" bottom (unrestrained condition)

CALCULATIONS

[A] Establish Slab Thickness

From PTI Manual the span/depth ratio for 2 way slabs is typically taken as 45 (Table 9.3)

$L/D = 45$
 $D = L/45 = 26 \times 12 / 45 = 6.9" \rightarrow 7" \text{ slab}$

Fig. : PARTIAL PLAN

Fig. : PARTIAL PLAN

Section A

TWO WAY PT SLAB SYSTEM - Example

[B] CALCULATE LOADING ON SLAB

Slab weight = 88 psf
 SDL = 20 psf
 Total Dead Load (DL) = 108 psf
 Live Load (LL) = 40 psf
 Reduce Live Load as per IBC

Spans 1 & 3

$R = 0.08 \cdot (A - 150) / 100 = 0.08 \cdot (396 - 150) / 100 = 0.2 \rightarrow 20\%$ controls [A = 18*22 = 396 ft²]
 $R_{max} = 23.1 \cdot (1 + DL/LL) = 23.1 \cdot (1 + 108/40) = 85\%$
 Reduced Live Load = [1 - 0.2] * 40 = 32 psf
 Factored DL = 1.2 * 108 = 130 psf
 Factored LL = 1.6 * 32 = 51 psf
 Total Unfactored Load; w = 140 psf
 Total Factored Load; w_u = 181 psf

Span 2

$R = 0.08 \cdot (A - 150) / 100 = 0.08 \cdot (572 - 150) / 100 = 0.33 \rightarrow 33\% < R_{max}$ [A = 26*22 = 572 ft²]
 Reduced Live Load = [1 - 0.33] * 40 = 27 psf
 Factored DL = 1.2 * 108 = 130 psf
 Factored LL = 1.6 * 27 = 43 psf
 Total Unfactored Load; w = 135 psf
 Total Factored Load; w_u = 173 psf

TWO WAY PT SLAB SYSTEM - Example

STEPS FOR DESIGN

- Select a percentage of Dead Load to be balanced by parabolic PT tendons
- Analyze an Equivalent Frame subjected to net downward loads (Net Load = Total Load – Balanced Load)
- Check flexural stresses at critical sections such as supports & mid-spans
- Revise PT tendons as required to obtain flexural stresses within limits (F_e or Drape)

- Calculate frame moments for factored loads
- Calculate secondary moments induced by PT forces
- Combine the two moments to obtain design factored moments
- Provide minimum bonded reinforcement as per ACI Code
- Compute design flexural strength of PT member considering both PT and non-PT reinforcement
- Increase non-PT reinforcement to satisfy factored moments if necessary

- Compute shear stresses due to vertical loads and moment transfer
- Compare to permissible shear values as per ACI Code
- Check deflection (normally not critical in PT slabs as the upward force of PT balances a good portion of the DL)

TWO WAY PT SLAB SYSTEM - Example

[C] LOAD BALANCING

Tendons are normally designed to balance 70 – 90% of the Dead Load of the structure.
Assume 80% Load Balancing with a parabolic tendon profile

Max. tendon sag in Span 2: $a = 7 - 1 - 1 = 5''$ [Effective Cover = Clear Cover + ¼"]

Prestress Force required to balance 80% of DL

$$F_e = [w_{bal} * L^2 / 8a] = [(0.8 * 0.088) * (26^2) * 12 / (8 * 5)] = 14.2 \text{ k/ft}$$

Area of strand; $A = 0.153 \text{ in}^2$

Assume losses = 15 ksi

$$\text{Effective force per tendon; } f_e = A * [(0.7 * f_{pu}) - 15] = 0.153 * [(0.7 * 270) - 15] = 26.6 \text{ kips}$$

Number of tendons required for a 22' wide bay

$$N = 22' * 14.2 / 26.6 = 11.7 \rightarrow 12 \text{ tendons}$$

$$\text{With 12 tendons the Effective force per foot of slab; } F_e = 12 * 26.6 / 22 = 14.5 \text{ k/ft}$$

Compressive force on the concrete section; $f_{pc} = F_e / A = 14.5 / (7 * 12) = 0.173 \text{ ksi}$

For an effective prestress force of 14.5 k/ft, we can work out the Actual Balanced Load in Span 2:

$$w_{bal} = [8 * F_e * a / L^2] = [8 * 14.5 * 5 / (12 * 26^2)] = 0.072 \text{ ksf}$$

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TWO WAY PT SLAB SYSTEM - Example

[C] LOAD BALANCING – contd.

If we want to provide same balance load in all spans then the tendons need to be adjusted in Spans 1 & 3 as follows:

$$a = w_{bal} * L^2 / (8 * F_e) = 0.072 * 18^2 * 12 / (8 * 14.5) = 2.4''$$

$$\text{Midspan cgs} = (3.5 + 6) / 2 - 2.4 = 2.35'' \sim 2.375''$$

$$\text{Actual sag in spans 1 \& 3} = (3.5 + 6) / 2 - 2.375 = 2.375''$$

Actual balanced load in Spans 1 & 3:

$$w_{bal} = [8 * F_e * a / L^2] = [8 * 14.5 * 2.375 / (12 * 18^2)] = 0.071 \text{ ksf}$$

[D] Tendon Profile

Net load which causes flexure = Total unfactored Load – Balanced Load

$$\text{Spans 1 \& 3: } w_{net} = 0.140 - 0.071 = 0.069 \text{ ksf}$$

$$\text{Span 2: } w_{net} = 0.135 - 0.072 = 0.063 \text{ ksf}$$

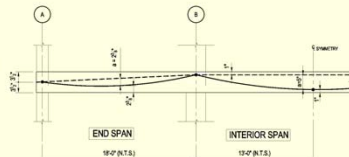
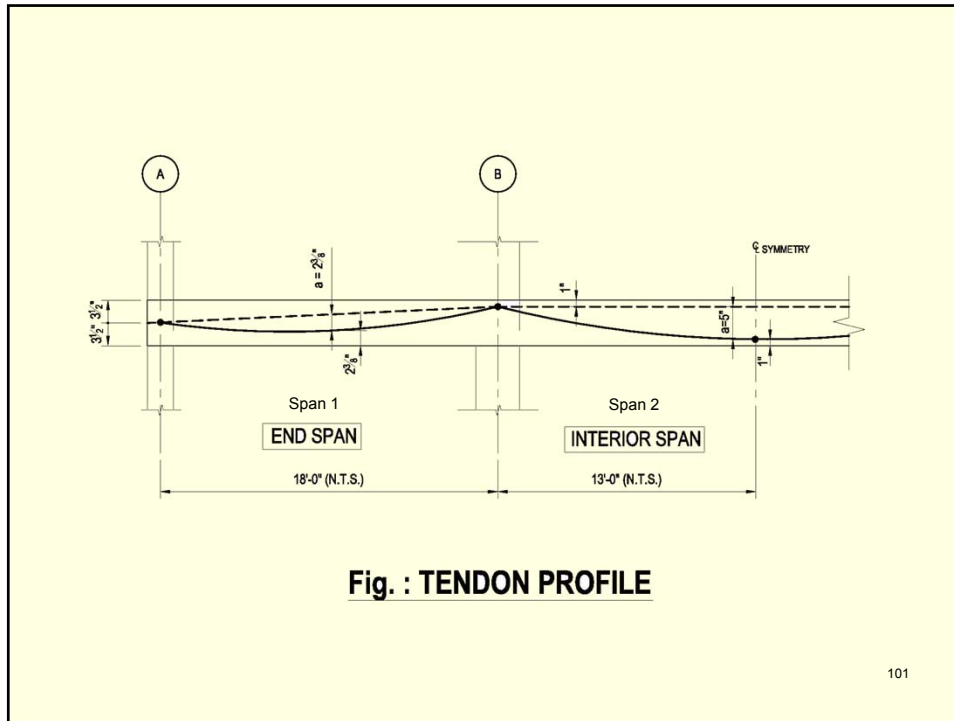


Fig.: TENDON PROFILE

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TWO WAY PT SLAB SYSTEM - Example

[E] EQUIVALENT FRAME PROPERTIES

From approx/simplified method in the PTI Manual, the column stiffness is computed from:

$K_c = 4EI / (L_c - 2h)$ where:

I = Inertia of the column, L_c = column height, h = slab thickness

Exterior Columns (16"x12" deep)

$$I = 16 \times 12^3 / 12 = 2304 \text{ in}^4$$

$$L_c = 9' = 108" \text{ and } h = 7"$$

E is a non-factor since the slab and column concrete strengths are same; so $E_c = E_s$ & $E_c/E_s = 1$

$$\text{Thus, } K_c = [4 \times 1 \times 2304 / (108 - 2 \times 7)] = 98 \text{ in}^3$$

For top and bottom columns

$$\Sigma K_c = 2 \times 98 = 196 \text{ in}^3 \text{ (joint total)}$$

Stiffness of torsional members:

$$C = (1 - 0.63 \times x/y) \times x^3 \times y / 3 = [1 - (0.63 \times 7/12)] \times 7^3 \times 12 / 3 = 868 \text{ in}^4$$

$$K_t = 9 \times C \times E_c / [L_2 \times (1 - c_2/L_2)^3] = 9 \times 868 \times 1 / [(22 \times 12) \times (1 - 1.33/22)^3] = 35.67 \text{ in}^3$$

$$\Sigma K_t = 2 \times 35.7 = 71.3 \text{ in}^3 \text{ (joint total)}$$

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TWO WAY PT SLAB SYSTEM - Example

[E] EQUIVALENT FRAME PROPERTIES – contd.

Exterior Equivalent Column Stiffness

$$K_{ec} = [1/ \sum K_t + 1/ \sum K_c]^{-1}$$

$$K_{ec} = [1/71.3 + 1/196]^{-1} = 52.3 \text{ in}^3$$

Interior Columns (16"x 20" deep)

Following a similar procedure as for Exterior column

$$I = 16 \cdot 20^3 / 12 = 10667 \text{ in}^4$$

$$K_c = [4 \cdot 1 \cdot 10667 / (108 - 2 \cdot 7)] = 454 \text{ in}^3$$

For top and bottom columns

$$\sum K_c = 2 \cdot 454 = 908 \text{ in}^3$$

Stiffness of torsional members:

$$C = (1 - 0.63 \cdot x/y) \cdot x^3 \cdot y/3 = [1 - (0.63 \cdot 7/20)] \cdot 7^3 \cdot 20/3 = 1782 \text{ in}^4$$

$$K_t = 9 \cdot C \cdot E_c / [L_2 \cdot (1 - c_2/L_2)^3] = 9 \cdot 1782 \cdot 1 / [(22 \cdot 12) \cdot (1 - 1.33/22)^3] = 73.2 \text{ in}^3$$

$$\sum K_t = 2 \cdot 73.2 = 146 \text{ in}^3 \text{ (joint total)}$$

Interior Equivalent Column Stiffness

$$K_{ec} = [1/ \sum K_t + 1/ \sum K_c]^{-1} = [1/146 + 1/908]^{-1} = 125.8 \text{ in}^3$$

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TWO WAY PT SLAB SYSTEM - Example

[E] EQUIVALENT FRAME PROPERTIES – contd.

Slab Beam Stiffness

Again from approx/simplified method the slab-beam stiffness can be calculated using:

$$K_s = 4EI / (L_1 - c_1/2)$$

where

I = Inertia of the slab system, L₁ = length of span in analysis direction, c₁ = column depth

At Exterior Columns

$$L_1 = 18' \text{ and } c_1 = 12'$$

$$K_s = 4 \cdot 1 \cdot 22 \cdot 7^3 / [(18 \cdot 12) - 12/2] = 143.7 \text{ in}^3$$

At Interior Columns (Spans 1 & 3)

$$K_s = 4 \cdot 1 \cdot 22 \cdot 7^3 / [(18 \cdot 12) - 20/2] = 146.5 \text{ in}^3$$

At Interior Columns (Span 2)

$$K_s = 4 \cdot 1 \cdot 22 \cdot 7^3 / [(26 \cdot 12) - 20/2] = 100 \text{ in}^3$$

Distribution Factors for Slab analysis

$$\text{Exterior Joint} = 143.7 / (143.7 + 52.3) = 0.73$$

$$\text{At interior Joint, spans 1 \& 3} = 146.5 / (146.5 + 100 + 125.8) = 0.39$$

$$\text{At interior Joint, span 2} = 100 / (146.5 + 100 + 125.8) = 0.27$$

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TWO WAY PT SLAB SYSTEM - Example

[F] MOMENT DISTRIBUTION using Net Loading

Fixed End Moment for uniform loads: $FEM = wL^2 / 12$

From Previous page

Span 2: $w_{net} = 0.140 - 0.071 = 0.063 \text{ ksf} \rightarrow FEM = 0.063 * 26^2 / 12 = 3.55 \text{ ft-k}$

Spans 1 & 3: $w_{net} = 0.135 - 0.072 = 0.069 \text{ ksf} \rightarrow FEM = 0.069 * 18^2 / 12 = 1.86 \text{ ft-k}$

Since $LL < 3/4 DL \rightarrow$ Pattern loading is not required. Place full LL on all spans

TABLE 1: MOMENT DISTRIBUTION - NET LOADS

D.F.	0.73	0.39	0.27	0.27	0.39
FEM	-1.86	-1.86	-3.55	-3.55	
DIST.	+1.36	-0.66	+0.46	+0.46	
CO.	+0.33	-0.68	-0.23	-0.23	
DIST.	-0.24	+0.17	-0.12	-0.12	
SUM	-0.41	-3.03	-3.44	-3.44	

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TWO WAY PT SLAB SYSTEM - Example

[G] CHECKING STRESSES FOR NET LOADS ON SPANS

Allowable Tension = $6 \sqrt{f'_c} = 6 \sqrt{5000} = +0.424 \text{ ksi}$

Allowable Comp. = $0.6f'_c = 0.6 * 5000 = -3.0 \text{ ksi (total load)}$

Allowable Comp. = $0.45f'_c = 0.45 * 5000 = -2.25 \text{ ksi (sustained load)}$

a] Stresses at inside face of Interior column

Calculate Moment at column using equation:

$M = -M_{CL} + V.c_1 / 3 \rightarrow$ Continuity in Frames, PCA, 1986

$M = -3.44 + [(0.063 * 26^2 / 2 * 20 / 12) / 3] = -3.44 + 0.455 = -2.985 \text{ ft-k}$

$S = bh^2 / 6 = 12 * 7^2 / 6 = 98 \text{ in}^3$

$f_t = -P/A + M/S = -0.173 + 12 * 2.985 / 98 = -0.173 + 0.367 = +0.194 \text{ ksi (T)} < +0.424 \text{ ksi}$

$f_b = -P/A - M/S = -0.173 - 12 * 2.985 / 98 = -0.173 - 0.367 = -0.54 \text{ ksi (C)} < -3.0 \text{ ksi}$

b] Stresses at mid-span of Span 2

$M = + [(0.063 * 26^2 / 8 - 3.55)] = +1.77 \text{ ft-k}$

$f_b = -P/A + M/S = -0.173 + 12 * 1.77 / 98 = -0.173 + 0.217 = +0.044 \text{ ksi (T)} < +0.424 \text{ ksi}$

$f_t = -P/A - M/S = -0.173 - 12 * 1.77 / 98 = -0.173 - 0.217 = -0.39 \text{ ksi (C)} < -3.0 \text{ ksi}$

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TWO WAY PT SLAB SYSTEM - Example

Note that when the tensile stress exceeds $2\sqrt{f_c}$ in positive moment areas, the total tensile force (N_c) must be carried by bonded reinforcement.

Check: $2\sqrt{f_c} = 2\sqrt{5000} = 0.144 > 0.044 \text{ ksi} \rightarrow$ Bonded reinf. not required

Fig. : SECTION

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TWO WAY PT SLAB SYSTEM - Example

DEFLECTIONS:

Larger span of 26' controls

As per ACI [18.3.3] PT two way slab systems shall be designed as Class U (Uncracked) w/ $f_t \leq 6\sqrt{f_c}$. The deflections can be calculated using elastic methods and gross concrete section properties. Note that deflections will be less because the PT force balances a substantial part of the dead load of the structure and the deflection is only caused by the net load.

See output from ADAPT and RAM-Concept for deflection results

[H] FLEXURAL STRENGTH

Design moments for indeterminate PT members are determined by combining frame moments due to factored DL & LL with secondary moments induced into the frame by tendons.

The balanced load moment includes both primary and secondary moments and as such secondary moments can be found by the following relation:

Secondary Moment = Balanced Load Moment – Primary Moment

$$M_2 = M_{bal} - M_1$$

From before,

Balanced Load in Span 2:	W _{bal} = 0.072 ksf;	FEM = 0.072 * 26 ² /12 = 4.06 ft-k	108
Balanced Load in Span 1 & 3:	W _{bal} = 0.071 ksf;	FEM = 0.071 * 18 ² /12 = 1.92 ft-k	

TWO WAY PT SLAB SYSTEM - Example

[H] FLEXURAL STRENGTH - contd

Again, $M_2 = M_{bal} - M_1$

where: $M_1 = P \cdot e$ and M_{bal} is obtained from moment distribution analysis as shown below
 $P = F_e = \text{Effective Prestress Force}$
 $e = \text{dist. Between cgs \& cgcs} = \text{eccentricity of PT}$

Thus the secondary moments are:

At Exterior Column: $M_2 = 0.41 - (14.5 \cdot 0/12) = +0.41 \text{ ft-k}$

At Interior Column: $M_2 = +3.29 - 14.5(3.5 - 1)/12 = +0.27 \text{ ft-k} \rightarrow \text{Spans 1 \& 3}$

At Interior Column: $M_2 = +3.88 - 14.5(3.5 - 1)/12 = +0.86 \text{ ft-k} \rightarrow \text{Span 2}$

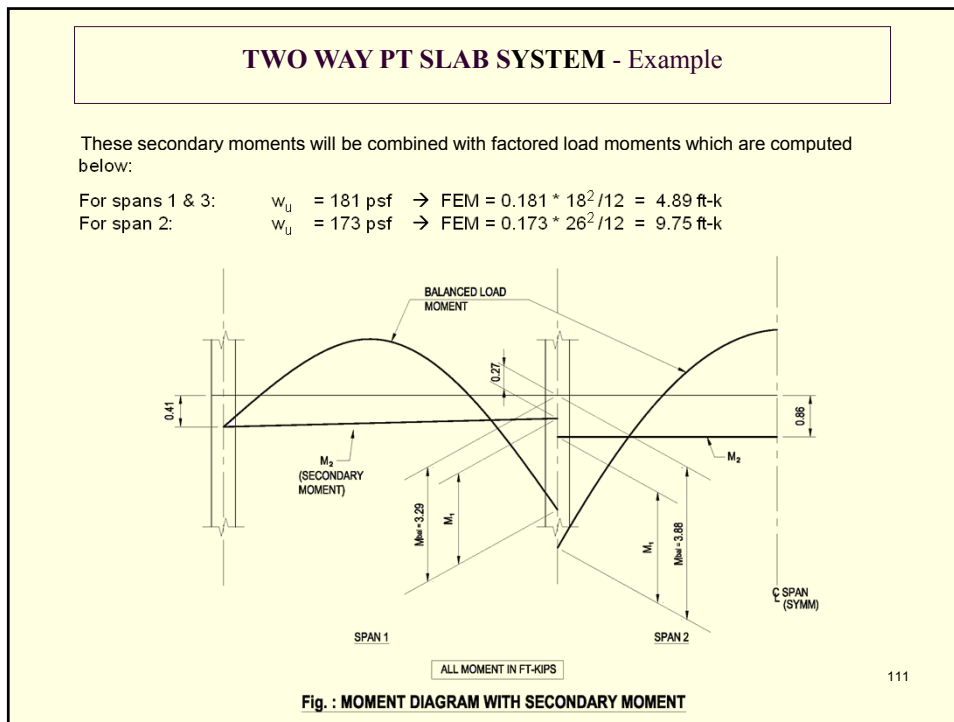
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TWO WAY PT SLAB SYSTEM - Example

TABLE 2: MOMENT DISTRIBUTION - BALANCED LOADS

D.F.	0.73	0.39	0.27	0.27	0.39
FEM	+1.92	+1.92	+4.06	+4.06	
DIST.	-1.40	+0.83	-0.58	-0.58	
CO.	-0.42	+0.70	+0.29	+0.29	
DIST.	+0.31	-0.16	+0.11	+0.11	
SUM	+0.41	+3.29	+3.88	+3.88	

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TWO WAY PT SLAB SYSTEM - Example

TABLE 3: MOMENT DISTRIBUTION - FACTORED LOADS

D.F.	0.73	0.39	0.27	0.27	0.39
FEM	-4.89	-4.89	-9.75	-9.75	
DIST.	+3.56	-1.89	+1.31	+1.31	
CO.	+0.95	-1.78	-0.65	-0.65	
DIST.	-0.69	+0.44	-0.31	-0.31	
SUM	-1.07	-8.12	-9.40	-9.40	

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TWO WAY PT SLAB SYSTEM - Example

Design Negative Moments at Face of Column are given in the Table below:

Total Positive Design Moments in the Spans are calculated as follows:

Span 1:

$$V_{ext} = [(0.181 * 18/2) - (7.85 - 0.66)] / 18 = 1.63 - 0.4 = 1.23 \text{ kip/ft}$$

	Span 1		Span 2
Factored load moments	-1.07	-8.12	-9.4
Secondary moments	0.41	+0.27	+0.86
Moments at column centerline	-0.66	-7.85	-8.54
Moment reduction by $V_c/3$	+0.54	+0.91	+1.25
Design moments at face of column	-0.12	-6.94	-7.29

Table 4: Design Moments at Face of Column (ft-kips)

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TWO WAY PT SLAB SYSTEM - Example

[H] FLEXURAL STRENGTH - contd.

$$V_{int} = 1.63 + 0.40 = 2.03 \text{ kip/ft}$$

Point of zero shear (max. moment): $x = 1.23/0.181 = 6.8'$

$$M_{max} = (0.5 * 1.23 * 6.81) - 0.66 = 3.52 \text{ ft-k}$$

Span 2:

$$V = 0.173 * 26/2 = 2.25 \text{ kip/ft}$$

$$M_{max} = -8.54 + (0.5 * 2.25 * 13) = 6.09 \text{ ft-k/ft}$$

FLEXURAL STRENGTH CALCULATIONS

Interior Column

ACI [18.9.3.3] stipulates a minimum amount of bonded top reinforcement at columns for ductility and crack control. More than minimum amount may be required to satisfy strength demands. The min amount is given by:

$$A_s = 0.00075 * A_{cf}$$

where:

A_{cf} = larger x-sect area of the slab-beam strips of the two perp equivalent frames intersecting at a column

$$A_s = 0.00075 * 7 * (18 + 26) / 2 * 12 = 1.386 \text{ in}^2$$

Use 7 #4 bars at spacing of 6" → Spread B = 6*6 = 3'-0" < 16 + 3 * 7 = 37" = 3'-1" OK

Bar Length = [2 * (26 - 20/12)/6] + 20/12 = 9.78' ~ 10'

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TWO WAY PT SLAB SYSTEM - Example

[H] FLEXURAL STRENGTH - contd.

For one foot strip:

$$A_s = 7 * 0.20 / 22 = 0.064 \text{ in}^2/\text{ft}$$

Stress in tendon at nominal strength:

$$f_{ps} = f_{se} + 10,000 + f'_c / (300 * \rho_p) \rightarrow \text{for } L/D > 35$$

With 12 tendons in 22' bay

$$\rho_p = A_{ps} / (b * d_p) = 12 * 0.153 / (22 * 12 * 6) = 0.00116 \text{ where: } d_p = (7 - 0.75 - 0.5/2) = 6"$$

$$f_{se} = (0.7 * 270) - 15 = 174 \text{ ksi}$$

$$f_{ps} = 174 + 10 + 5 / (300 * 0.00116) = 174 + 10 + 14.4 = 198.4 \text{ ksi}$$

$$f_{ps} \text{ not to be greater than } 0.85 * f_{pu} = 230 \text{ ksi} \rightarrow \text{OK}$$

$$f_{ps} \text{ not to be greater than } f_{se} + 30 = 204 \text{ ksi} \rightarrow \text{Ok}$$

$$A_{ps} * f_{ps} = 12 * 0.153 * 198.4 / 22 = 16.56 \text{ k/ft}$$

$$A_s * f_y = 0.064 * 60 = 3.84 \text{ k/ft}$$

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TWO WAY PT SLAB SYSTEM - Example

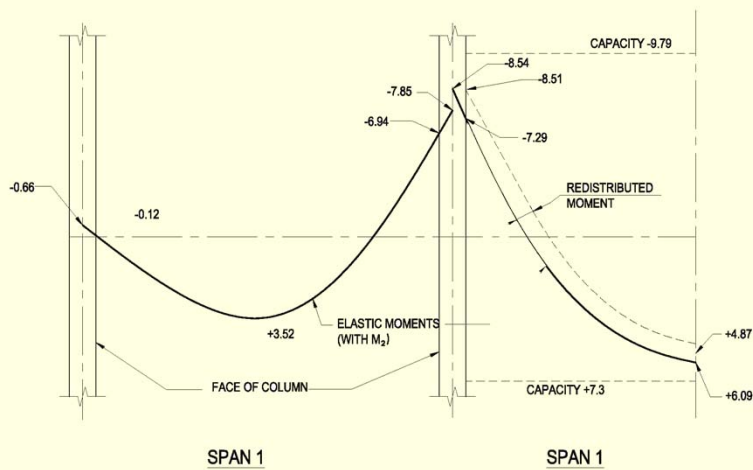


Fig. : DESIGN MOMENTS (FT-KIPS)

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TWO WAY PT SLAB SYSTEM - Example

[H] FLEXURAL STRENGTH - contd.

$$a = [(A_{ps} * f_{ps}) + (A_s * f_y)] / (0.85 f_c * b) = [16.56 + 3.84] / (0.85 * 5 * 12) = 0.4"$$

$$c = a / \beta_1 = 0.4 / 0.8 = 0.47 \rightarrow \beta_1 = 0.8 \text{ for } f_c = 5000 \text{ psi}$$

From the strain diagram: $\epsilon_t = (6 - 0.47) * 0.003 / 0.47 = 0.035 > 0.005$
 \rightarrow Tension Controlled; Use $\phi = 0.9$

For both the tendon and the rebar (since they are in the same layer)

$$(d - a/2) = (6 - 0.4/2) / 12 = 0.48'$$

$$\phi M_n = (0.9 * (16.56 + 3.84) * 0.48 = 9.79 > 7.29 \text{ ft-k/ft} \quad \text{OK}$$

Moment Redistribution can be used to reduce the positive moment demand in Span 2 as there is excess negative moment capacity available.

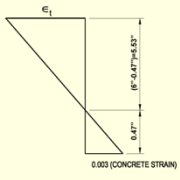
Midspan of Span 2: $a = 16.56 / (0.85 * 5 * 12) = 0.325"$

$$c = a / \beta_1 = 0.325 / 0.8 = 0.41"$$

$$\epsilon_b = 5.62 * 0.003 / 0.40 = 0.042$$

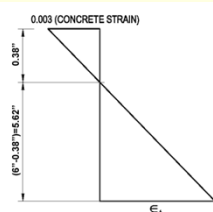
Allowed Reduction in positive moment = $1000 \epsilon_b = 1000 * 0.042 = 42\% > 20\%$

Available decrease in Positive Moment = $0.2 * 6.09 = 1.22 \text{ ft-k/ft}$



AT INTERIOR SUPPORT

Fig.: STRAIN DIAGRAM



AT MIDSPAN (SPAN 2)

TWO WAY PT SLAB SYSTEM - Example

[H] FLEXURAL STRENGTH - contd.

Increased Negative Moment = $7.29 + 1.22 = 8.51 \text{ ft-k/ft} < 9.79 \text{ OK}$
 Design Positive Moment in Span 2 = $6.09 - 1.22 = 4.87 \text{ ft-k/ft}$

Since tension stresses $< 2 \sqrt{f_c} \rightarrow$ No bonded reinforcement required

Capacity at mid-span of Span 2:

$$A_{ps} * f_{ps} = 12 * 0.153 * 198.4 / 22 = 16.56 \text{ k/ft}$$

$$a = 16.56 / (0.85 * 5 * 12) = 0.325"$$

$$c = a / \beta_1 = 0.325 / 0.8 = 0.406"$$

$$c/d_t = 0.406/6 = 0.068 < 0.375 \rightarrow$$
 Tension Controlled; Use $\phi = 0.9$

$$(d - a/2) = (6 - 0.325/2) / 12 = 0.49'$$

$$\phi M_n = (0.9 * (16.56) * 0.49 = 7.3 > 4.87 \text{ ft-k/ft} \quad \text{OK} \quad \text{[Mid-span]}$$

Capacity at mid-span of Span 1:

$$(d - a/2) = [(7 - 2.375) - 0.325/2] / 12 = 0.37'$$

$$c/d_t = 0.325/0.8/4.625 = 0.083 < 0.375 \rightarrow$$
 Tension Controlled; Use $\phi = 0.9$

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TWO WAY PT SLAB SYSTEM - Example

[H] FLEXURAL STRENGTH - contd

$$\phi M_n = (0.9 * (16.56)) * 0.37 = 5.51 > 3.52 \text{ ft-k/ft OK [Mid-span]}$$

EXTERIOR COLUMNS

$$A_s \text{ min} = 0.00075 * 7 * (22 * 12) = 1.39 \text{ in}^2 \rightarrow \text{Use 7 \#4 bars}$$

For one foot strip:

$$A_s = 7 * 0.20 / 22 = 0.064 \text{ in}^2/\text{ft}$$

$$A_s * f_y = 0.064 * 60 = 3.84 \text{ k/ft}$$

With 12 tendons in 22' bay

$$\rho_p = A_{ps} / (b * d_p) = 12 * 0.153 / (22 * 12 * 3.5) = 0.002 \text{ where: } d_p = 3.5" \text{ at exterior column}$$

$$f_{ps} = f_{se} + 10,000 + f'_c / (300 * \rho_p)$$

$$= 174 + 10 + 5 / (300 * 0.002) = 192.3 \text{ ksi}$$

$$A_{ps} * f_{ps} = 12 * 0.153 * 192.3 / 22 = 16.05 \text{ k/ft}$$

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TWO WAY PT SLAB SYSTEM - Example

[H] FLEXURAL STRENGTH - contd.

$$a = [16.05 + 3.82] / (0.85 * 5 * 12) = 0.39"$$

$$c = a / \beta_1 = 0.39 / 0.8 = 0.46"$$

$$\epsilon_t = (6 - 0.46) * 0.003 / 0.46 = 0.036 \rightarrow \text{Tension Controlled; Use } \phi = 0.9$$

Tendons:

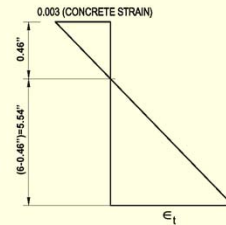
$$(d_p - a/2) = (3.5 - 0.39/2) / 12 = 0.28'$$

Rebars:

$$(d - a/2) = (6.0 - 0.39/2) / 12 = 0.484'$$

$$\phi M_n = 0.9 * [(16.05 * 0.28) + (3.84 * 0.484)] = 5.71 > 0.12 \text{ ft-k/ft OK}$$

This concludes the computation of the flexural strength at mid-span and at the column for the 2 way slab system.



AT MIDSPAN (SPAN 1)

Fig. : STRAIN DIAGRAM

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TWO WAY PT SLAB SYSTEM - Example

[J] SHEAR AT EXTERIOR COLUMNS

a] Shear: $V_u = [(0.181 * 18/2) - (7.85 - 0.66)]/18 = 1.23 \text{ k/ft}$

Assume cladding line load; $w = 0.35 \text{ k/ft}$

Total $V_u = [(1.2 * 0.35) + 1.23] * 22 = 36.3 \text{ kips}$

Transfer Moment = $22 * 0.66 = 14.5 \text{ ft-kips}$

where: factored moment at ext column from Table 4 = 0.66 ft-kips/ft

b] Combined direct & eccentric shear at inside face of critical transfer section

$$v_u = (V_u / A) + (\gamma_v * M_u * c / J)$$

where:

$d = 0.8 * 7 = 5.6''$

$C_1 = 12''$

$C_2 = 16''$

$b_1 = C_1 + 5.6/2 = 14.8''$

$b_2 = C_2 + 5.6 = 21.6''$

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TWO WAY PT SLAB SYSTEM - Example

[J] SHEAR AT EXTERIOR COLUMNS – cont'd.

$c = b_1^2 / (2 * b_1 + b_2) = 4.28''$

$A_c = (2b_1 + b_2) * d = 286.7 \text{ in}^2$

$J/c = [2b_1d(b_1 + 2b_2) + d^3(2b_1 + b_2)/b_1] / 6 = 1703.5 \text{ in}^3$

$\gamma_f = 1 / [1 + (2/3 * \sqrt{(b_1/b_2)})] = 0.64$

$\gamma_v = 1 - \gamma_f = 1 - 0.64 = 0.36$

$v_u = (V_u / A) + (\gamma_v * M_u * c / J)$
 $= [(36.3/286.7) + (0.36 * 14.5 * 12000 / 1703.5)] = 126.6 + 36.8 = 163.4 \text{ psi}$

c] Allowable Shear Stress as per ACI

$\phi v_n = \phi V_c / (b_o d) = \phi * 4 \sqrt{f'_c} \rightarrow \text{for Edge columns}$

$= 0.75 * 4 \sqrt{5000} = 212 \text{ psi} > 163.4 \text{ psi OK}$

d] Checking Moment Transfer

Effective slab width for this moment transfer = width of column + $2(1.5 * \text{slab thickness})$

$= 16 + 2(1.5 * 7) = 37 \text{ in}^2$

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TWO WAY PT SLAB SYSTEM - Example

[J] SHEAR AT EXTERIOR COLUMNS – contd.

Assume 3 tendons are bundled together and go thru the column
 Min amount of bonded reinforcement from before: $A_s = 0.00075 A_{cf} = 1.39 \text{ in}^2 \rightarrow 7\#4 \text{ bars}$

$$\rho_p = 3 * 0.153 / (37 * 3.5) = 0.00354$$

$$f_{ps} = f_{se} + 10,000 + f'_c / (300 * \rho_p)$$

$$= 174 + 10 + 5 / (300 * 0.00354) = 188.7 \text{ ksi}$$

$$\text{Corresponding PT force} = A_{ps} * f_{ps} = 3 * 0.153 * 188.7 = 86.6 \text{ kips}$$

$$A_s * f_y = 7 * 0.20 * 60 = 84 \text{ kips}$$

$$(A_{ps} * f_{ps}) + (A_s * f_y) = 86.6 + 84 = 170.6 \text{ kips}$$

$$a = 170.6 / (0.85 * 5 * 37) = 1.09''$$

$$\text{Tendon } (d_p - a/2) = (3.5 - 1.09/2) / 12 = 0.246'$$

$$\text{Rebar } (d - a/2) = (6 - 1.09/2) / 12 = 0.45'$$

$$\phi M_n = (0.9 * (86.6 * 0.246) + (84 * 0.45)) = 53.2 \text{ ft-kips}$$

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TWO WAY PT SLAB SYSTEM - Example

[J] SHEAR AT EXTERIOR COLUMNS – contd.

$$Y_f = 0.64$$

$$Y_f M_u = 0.64(14.5) = 9.28 \text{ ft-kips} \ll 53.2 \text{ ft-kips OK}$$

[K] SHEAR AT INTERIOR COLUMNS

a] Shear & Transfer Moment

$$V_u = [(2.03 + 2.25) * 22] = 94.2 \text{ kips}$$

$$M_u = 22 * (8.54 - 7.85) = 15.18 \text{ ft-kips}$$

b] Combined direct & eccentric shear at inside face of critical transfer section

$$v_u = (V_u / A) + (\gamma_v * M_u * c / J)$$

where:

$$d = 0.8 * 7 = 5.6''$$

$$C_1 = 20''$$

$$C_2 = 16''$$

$$b_1 = C_1 + d = 25.6''$$

$$b_2 = C_2 + d = 21.6''$$

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TWO WAY PT SLAB SYSTEM - Example

[K] SHEAR AT INTERIOR COLUMNS – contd.

$$A_c = 2(b_1 + b_2) * d = 528.6 \text{ in}^2$$

$$J/c = [b_1 d (b_1 + 3b_2) + d^3] / 3 = 4378 \text{ in}^3$$

$$\gamma_f = 1 / [1 + (2/3 * \sqrt{b_1/b_2})] = 0.58$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.58 = 0.42$$

$$v_u = (V_u / A) + (\gamma_v * M_u * c / J) = [94.2 / 528.6] + (0.42 * 15.18 * 12 / 4378) = 178 + 17.5 = 195.5 \text{ psi}$$

c] Allowable Shear Stress as per ACI

$$\phi v_c = \phi [\beta_p \lambda * \sqrt{f'_c} + (0.3 * f_{pc}) + V_p / (b_o * d)] \rightarrow \text{for Interior columns}$$

$$\beta_p = [\alpha_s * d / b_o + 1.5] \leq 3.5$$

where: $b_o = 2[(20 + 5.6) + (16 + 5.6)] = 94.4"$ and $\alpha_s = 40$ for interior columns

$$\beta_p = [40 * 5.6 / 94.4 + 1.5] = 3.87 > 3.5 \rightarrow \text{Use } \beta_p = 3.5$$

V_p is the component of shear capacity contributed by the tendons. It is normally neglected especially for thin slabs because field placement of tendons can greatly affect this value.

$$\text{Thus: } \phi v_c = 0.75 * [3.5 * \sqrt{5000} + (0.3 * 173)] = 224.5 \text{ psi} > 195.5 \text{ psi} \text{ OK}$$

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TWO WAY PT SLAB SYSTEM - Example

[K] SHEAR AT INTERIOR COLUMNS – contd.

d] Checking Moment Transfer

$$\text{Effective slab width for this moment transfer} = \text{width of column} + 2 (1.5 * \text{slab thickness}) = 16 + 2 (1.5 * 7) = 37 \text{ in}^2$$

From above: $\gamma_f = 0.58$

$$\text{Moment Transfer} = 0.58 * (15.18) = 8.8 \text{ ft-kips}$$

$$A_{ps} * f_{ps} = 86.6 \text{ kips (same as exterior column)}$$

$$A_s = 0.00075 A_{cf} = 0.00075 * 7 * (18 + 26) / 2 * 12 = 1.39 \text{ in}^2 \rightarrow \text{use 7\#4 bars}$$

$$A_s * f_y = 7 * 0.20 * 60 = 84 \text{ kips}$$

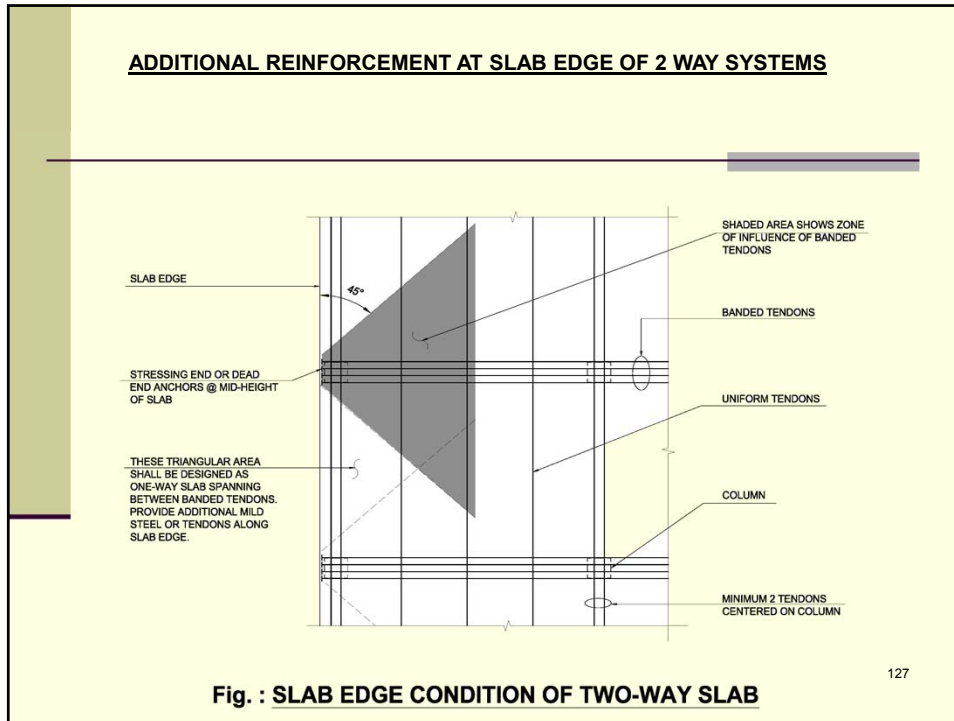
$$(A_{ps} * f_{ps}) + (A_s * f_y) = 86.6 + 84 = 170.6 \text{ kips}$$

$$a = 170.6 / (0.85 * 5 * 37) = 1.68"$$

$$(d - a/2) = (6 - 1.08 / 2) / 12 = 0.46'$$

$$\phi M_n = [0.9 * (170.6 * 0.46)] = 70.6 \text{ ft-kips} \gg 8.8 \text{ ft-kips} \text{ OK}$$

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SHEAR DESIGN OF PRESTRESSED MEMBERS

There are essentially two types of shear failures in prestressed beams:

1. Web-shear cracking, V_{cw} – Cracking starts in the web as a result of high principal stresses
2. Flexure shear cracking, V_{ci} – This is also known as Inclined Flexural cracking. In this type the vertical flexural cracks occur first and gradually develop into inclined shear

The cracks are shown (both for simple and continuous beams) in Fig 1 & 2 in the following pages.

The ACI Code stipulates that the concrete shear strength V_c shall be the lesser of V_{cw} and V_{ci} which shall be computed as shown below:

$$V_{cw} = (3.5\lambda \sqrt{f'_c} + 0.3 f_{pc}) b_w d_p + V_p \quad \rightarrow \text{ [ACI Eq. 11-12]}$$

where: d_p = effective depth of prestressing steel (0.8h min)
 f_{pc} = compressive stress in concrete after allowance for all losses at centroid of section
 λ = factor to account for Lt-wt concrete ($\lambda = 1$ for normal wt concrete)
 V_p = vertical component of effective prestress force at section

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SHEAR DESIGN OF PRESTRESSED MEMBERS – contd.

$$V_{ci} = 0.6\lambda \sqrt{f_c} b_w d_p + V_d + V_l M_{cre} / M_{max} \quad \rightarrow \text{[ACI Eq. 11-10]}$$

where: $M_{cre} = (I / y_t) (6 \lambda \sqrt{f_c} + f_{pe} - f_d)$ $\rightarrow \text{[ACI Eq. 11-11]}$

\rightarrow moment causing flexural cracking at section due to externally applied loads

f_{pe} = compressive stress in concrete after allowance for all losses due to effective prestress force only, at extreme fiber of section

f_d = stress at extreme fiber of section due to unfactored dead load

V_d = shear due to unfactored dead load at a section

M_{max} and V_l shall be computed from the load combination causing max factored moment to occur at the section.

Note: $V_{ci} \geq 1.7\lambda \sqrt{f_c} b_w d$

The V_{cw} and V_{ci} values for a hypothetical beam are plotted in figure 3. Note that shear stirrups are to be provided for the difference between the V_u / Φ and the V_c values.

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A simplified equation (11-9) is also provided by ACI 11.3.2 which is illustrated in Example 7.

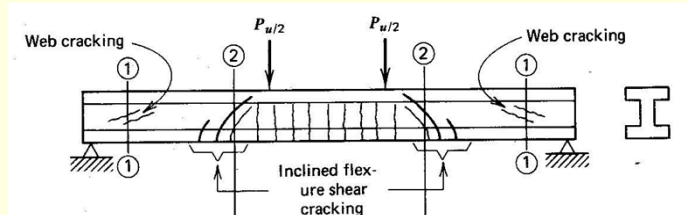


Fig. 1 – Shear cracks in Simple beam

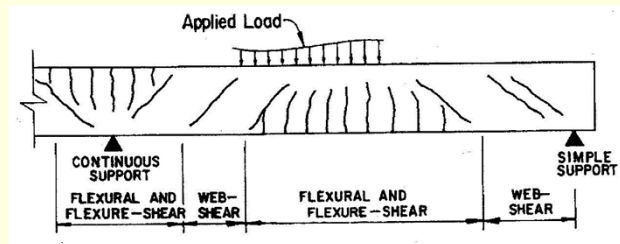
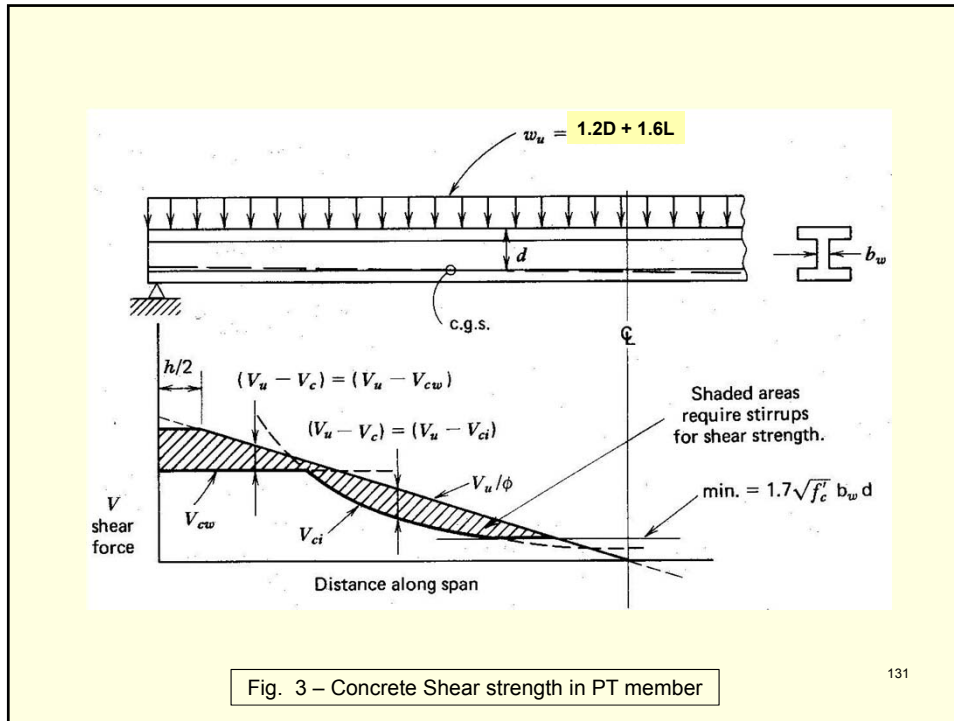


Fig. 2 – Shear cracks in Continuous beam

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SHEAR DESIGN OF PRESTRESSED BEAM

EXAMPLE – 7

For the beam in Example 4, check Shear design near support

Assume:
 Bonded Parabolic tendons
 Beam carries Lt-wt Precast Planks @ 60 psf
 NW Concrete for PT Beam
 SDL @ 20 psf (Partition/MEP)
 LL @ 65 psf
 Trib. Width B = 16'-0"

Design Data

$b_w = 14"$
 $h = 26"$
 $d = 23"$
 $f'_c = 5000 \text{ psi}$
 $f'_{pu} = 270,000 \text{ psi}$
 $L = 30' - 0"$

$w_D = \left(\frac{14 \times 26}{144}\right) 0.15 = 0.38 \text{ k/ft}$

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SHEAR DESIGN OF PRESTRESSED BEAM – cont.

$$w_{SD} = (60 + 20) \frac{16}{1000} = 0.08 \times 16 = 1.28 \text{ k/ft}$$

$$w_L = 65 \times \frac{16}{1000} = 1.04 \text{ k/ft}$$

$$w_S = 0.38 + 1.28 + 1.04 = 2.7 \text{ k/ft}$$

$$w_u = (0.38 + 1.28)1.2 + (1.04)1.6 = 3.66 \text{ k/ft}$$

$$M_u = (3.66 \times 30^2 / 8) = 412'k < \phi M_n = 414'k \rightarrow \text{For Bonded Tendons from Example 4}$$

Shear force @ dist. h/2 from pin support

$$V_u = w_u(L/2 - h/2) = 3.66 \left(\frac{30}{2} - \frac{13}{12} \right) = 50.9k$$

Using Simplified Approach as per Eq. 11-9 [ACI 11.3.2]

$$V_c = \left(0.6 \lambda \sqrt{f'_c} + 700 \frac{V_u d_p}{M_u} \right) b_w d \rightarrow \text{Eq (11-9)}$$

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SHEAR DESIGN OF PRESTRESSED BEAM – cont.

but,

$$V_c \geq 2 \lambda \sqrt{f'_c} b_w d = 2 \times 1 \times \sqrt{5000} \times 14 \times 23 = 45.5k$$

$$V_c \leq 5 \sqrt{f'_c} b_w d = 45.5 \times 5 / 2 = 113.8k$$

where:

$$d = 23''$$

$$d_p = 26 \times 0.8 = 20.8''$$

$$\lambda = 1 \text{ (NW Concrete)}$$

Note the above equation can only be used if: $f_{se} \geq 0.4 f_{pu}$ [which is correct as per Example 4]
Eq. (11-9) can be further simplified for simply supported member with uniform loads to:

$$V_c = \left\{ 0.6 \sqrt{f'_c} + 700 \left[d_p \frac{(l-2x)}{x(l-x)} \right] \right\} b_w d \quad \text{[ACI R11.3.2]}$$

where:

$$\frac{V_u d_p}{M_u} = \frac{(l-2x)}{x(l-x)} d_p \quad \text{(ACI R11.3.2)}$$

$$= \left[\frac{30 - 2(1.08)}{1.08(30 - 1.08)} \right] \times \frac{20.8}{12}$$

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SHEAR DESIGN OF PRESTRESSED BEAM – cont.

$$\frac{V_u d_p}{M_u} = 1.54 \nabla 1.0$$

≈ Use 1.0

where:

$$x = \frac{26/2}{12} = 1.08''$$

$$f'_c = 5000 \text{ psi}$$

$$V_C = [0.6\sqrt{5000} + 700(1)]14 \times 20.8$$

$$= \left[\frac{42.4 + 700}{1000} \right] (291.2)$$

$$= 216k$$

$$> 113.8k \text{ Use: } V_C = 113.8k$$

$$\phi V_C = 0.75 \times 113.8 = 85.4^k > V_u = 50.9^k$$

$$\frac{\phi V_C}{2} = \frac{85.4}{2} = 42.7^k < V_u$$

Minimum shear reinforcement is required

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SHEAR DESIGN OF PRESTRESSED BEAM – cont.

As per ACI 11.4.6.3:

$$Av_{min} = 0.75\sqrt{f'_c} \frac{b_w s}{f_{yt}} \geq \frac{50b_w s}{f_{yt}} \quad Eq (11 - 13)$$

$$= 0.75\sqrt{5000} \frac{(14 \times 12)}{60,000} \geq \frac{50 \times 14 \times 12}{60,000}$$

$$= 53 \times 0.0028 \geq 50 \times 0.0028 \text{ (does not control)}$$

$$= 0.148 \text{ in}^2 / f_t$$

where f_{yt} is the yield strength of transverse reinforcement

As per ACI 11.4.6.4

$$Av_{min} = \frac{A_{ps} f_{pu} \cdot s}{80 f_{yt} d} \sqrt{\frac{d}{b_w}} \quad Eq (11 - 14)$$

$$= \frac{0.153 \times 7 \times 270 \times 12}{80 \times 60 \times 20.8} \sqrt{\frac{20.8}{14}}$$

$$= 0.0348(1.22)$$

$$= 0.043 \text{ in}^2 / f_t$$

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SHEAR DESIGN OF PRESTRESSED BEAM – cont.

As per ACI, the lesser of the two values from Eq. (11-13) & Eq. (11-14) shall be used

$$\text{Max. stirrup spacing} = s = \frac{3}{4}d = \frac{3}{4} \times 20.8 = 15.6''$$

Use: #3@12''

$$A_v = 0.22 \text{ in}^2 > 0.043$$

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CRACK FORMATION**■ DEFICIENT SLAB SYSTEM**

- Inadequate design (engineering issue)
- Inadequate detailing (engineering or shop issue)
- Poor workmanship (construction issue)

■ RESTRAINT

- Restraint to shortening of slab system

The effect of restraint is covered in detail here

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RESTRAINT CRACK CAUSES

- PT Slabs tend to shorten – Elastic Shortening
- Restrained Volume Changes which include shrinkage, creep & temperature effects
- Walls and columns restrain free movement of slab
- Irregularities in slab geometry such as opening & re-entrant corners
- Tension developed in slab due to restraint exceeds tensile capacity of concrete
- Tensile capacity ~ 250 to 300 psi

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CONTRIBUTION OF DIFFERENT FACTORS TO SLAB SHORTENING

RANGE

- Shrinkage ~ 50 – 65%
- Creep + Elastic Shortening ~ 18 – 20%
- Temperature effects ~ 15 – 25%

FOR CHICAGO AREA

- Shrinkage ~ 60%
- Creep + ES ~ 20%
- Temp. ~ 20%

Axial Creep & Elastic Shortening which are direct consequence of PT are about 1/5th of total shortening; the balance of shortening is common to both PT & non-PT slabs

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CREEP & SHRINKAGE SHORTENING

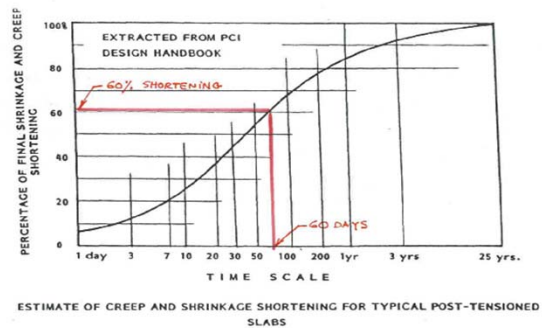


Figure 2.12

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CHARACTERISTICS OF CRACKS

PT CRACKS

- Fewer in number
- Wider & deeper
- Spaced out – $\frac{1}{4}$ of shorter slab span
- Do not generally coincide with location of max bending moments such as mid-span bottom fibers and top of supports
- Occur at axially weak locations such as construction joints, cold joints, pour strips, reduced pre-compression, section discontinuities etc.
- Bonded tendon slab exhibit better crack behavior than unbonded tendons
- Unbonded tendons may need more mild steel reinforcement for crack control

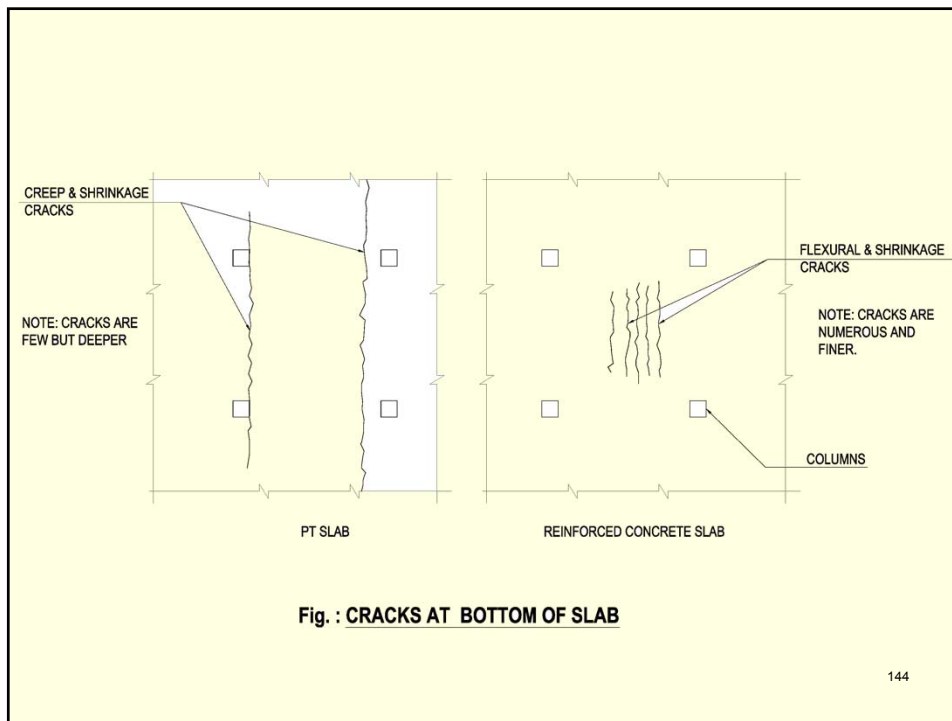
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CHARACTERISTICS OF CRACKS

CRACKS IN MILD STEEL FLOOR SYSTEMS

- Large number of hairline cracks
- Shallower cracks
- Crack spacing closer – equal to slab depth or less
- Cracks shorter in length
- Shrinkage, flexural or shear cracks
- Cracks coincide with location of max bending moments such as mid-span bottom fibers and top of supports

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CRACK MITIGATION

- A. Planning & Layout of restraining members (walls & columns)
- B. Structural Separation & Expansion Joints
- C. Pour Strips & Construction Joints
- D. Released Connections
- E. Addition & Improved layout of mild steel reinforcement
- F. Improved layout of PT tendons

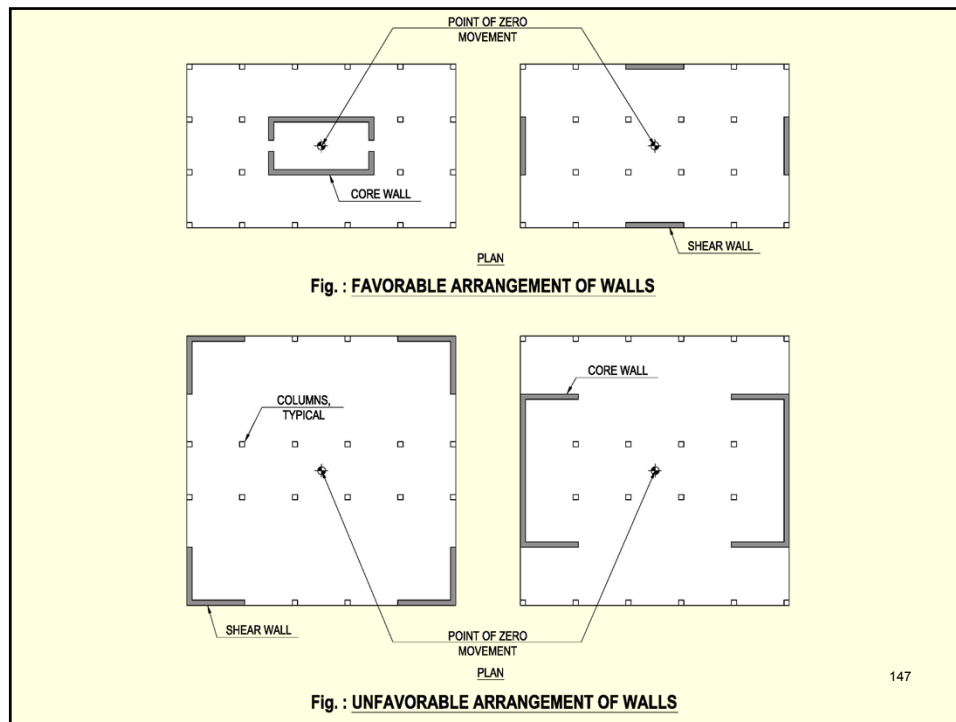
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CRACK MITIGATION

A] PLANNING & LAYOUT OF RESTRAINING ELEMENTS

- Layout of stiff vertical elements shall allow the slab to shrink and move to point of zero movement
- See favorable and unfavorable layouts

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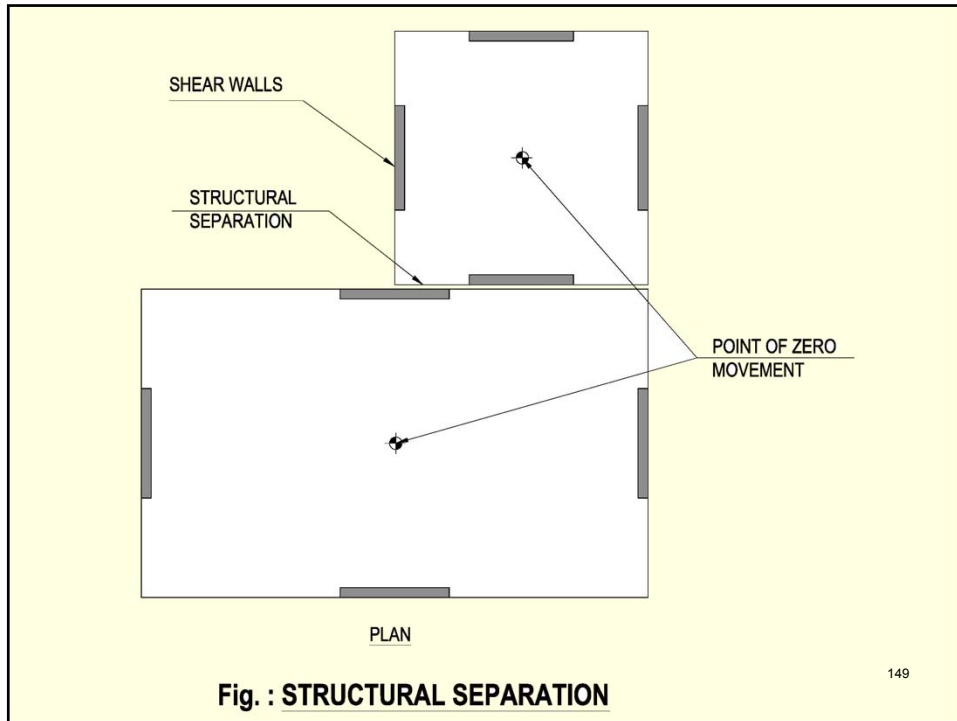
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CRACK MITIGATION

B] STRUCTURAL SEPARATION & EXPANSION JOINTS

- Introduce separation joint or expansion joint for irregular geometry of slab and appendages to slabs
- Structural Separation— ½" to 1" gap
temporary; filled later with concrete or grout typically after two to three months allows each portion of slab to shrink separately
- Expansion Joint – 1" to 3" wide
 - permanent; flexible material installed in joint
 - runs through the vertical support elements
 - edge columns each side of expansion joint
 - designed to be serviceable through the life of structure

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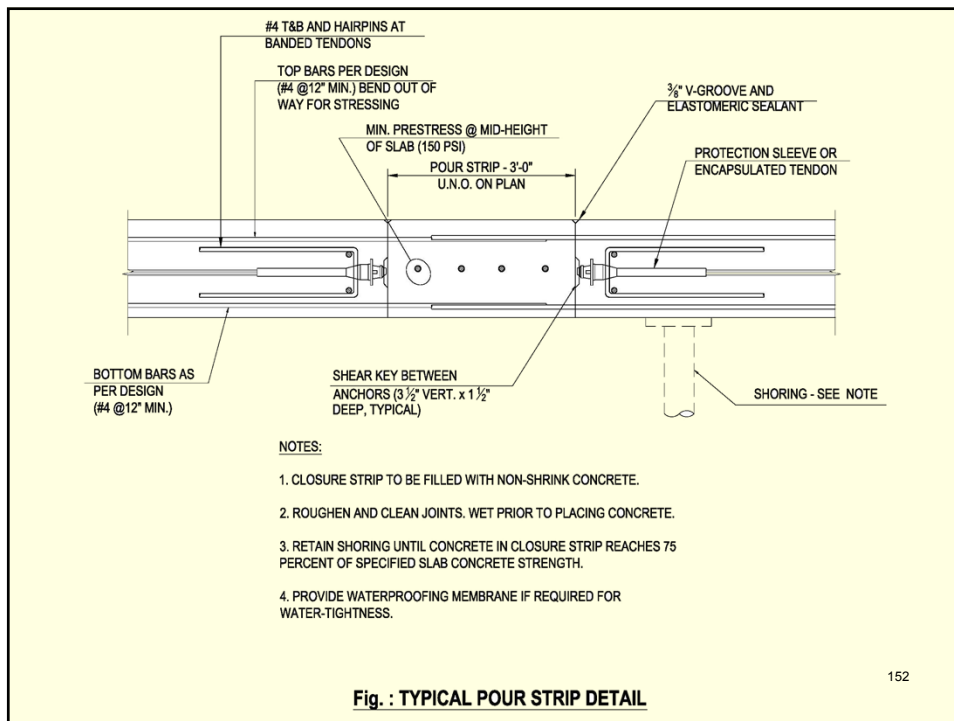


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POUR STRIPS & CONSTRUCTION JOINTS

C] POUR STRIP OR CLOSURE STRIP

- Normally 30" to 36" wide – should be enough for stressing operations
- Left open for about 60 days. Accounts for about 60% of the total shortening.
- Slab on each side of pour strip is constructed and post-tensioned separately
- Reinforcement designed on basis of moments & shears occurring at the location of the strip when the entire slab is combined into a continuum
- Reinforcement from each side shall overlap the other side and provide continuity
- Normally places at 1/5 to 1/4 span. The PT on the short span shall be designed as cantilever or the entire bay shall be designed for mild steel reinforcement.
- If the span is short compared to the adjacent spans on each side then the pour strip is normally placed at the center of the short span.



POUR STRIPS & CONSTRUCTION JOINTS

CONSTRUCTION JOINT

- Provides planned temporary break between two slab regions for crack control and construction operations
- Used to divide large slab area into manageable size
- Allows for shortening of slab due to volume change effects
- Normal time lag = 3 to 5 days
- CJ may or may not have intermediate stressing.
- Intermediate stressing is carried out for long tendons where friction losses are appreciable.

GUIDELINES FOR LOCATING POUR STRIPS & SEPARATION JOINTS

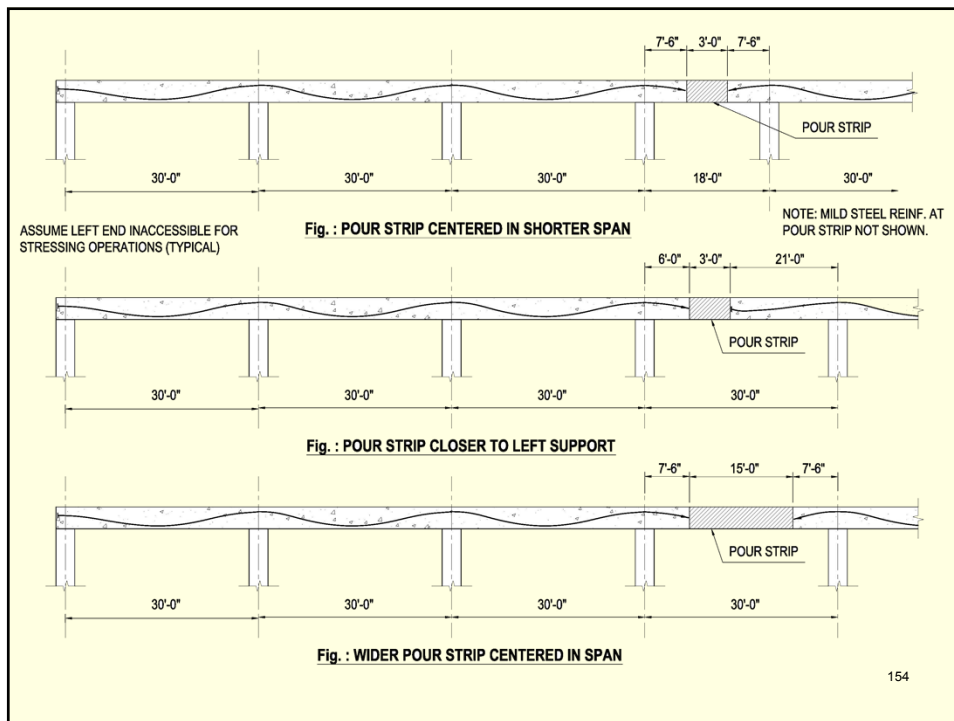
$L < 250'$ → No pour strip or separation needed

$250' < L < 375'$ → Provide one centrally located pour strip

$L > 375'$ → Provide structural separation such as expansion joint

The above guideline is assuming favorable location of columns/walls; otherwise the length provisions will be tighter.

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POUR STRIPS & CONSTRUCTION JOINTS

D] RELEASED CONNECTIONS

- These allow limited slip movement of slab over restraining elements.
- This is especially important when favorable layout of shearwall is not possible.
- Releases can be permanent or temporary
- Plastic film over smooth (trowel) finished wall before slab is poured.
- Dowels are provided between the wall and slab but these are not engaged to the slab.
- Temporary Release - After the slip movement has taken place the dowels are locked in place by filling the holes with grout.
- Permanent Release – The holes remain open as they are not filled with grout. The wall does not provide redundancy to the structure. The dowel helps against catastrophic failure.
- Examples of slip material – Neoprene or visqueen.

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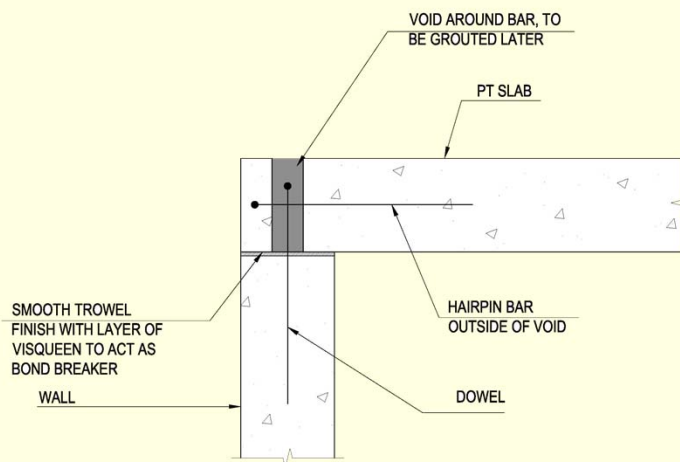
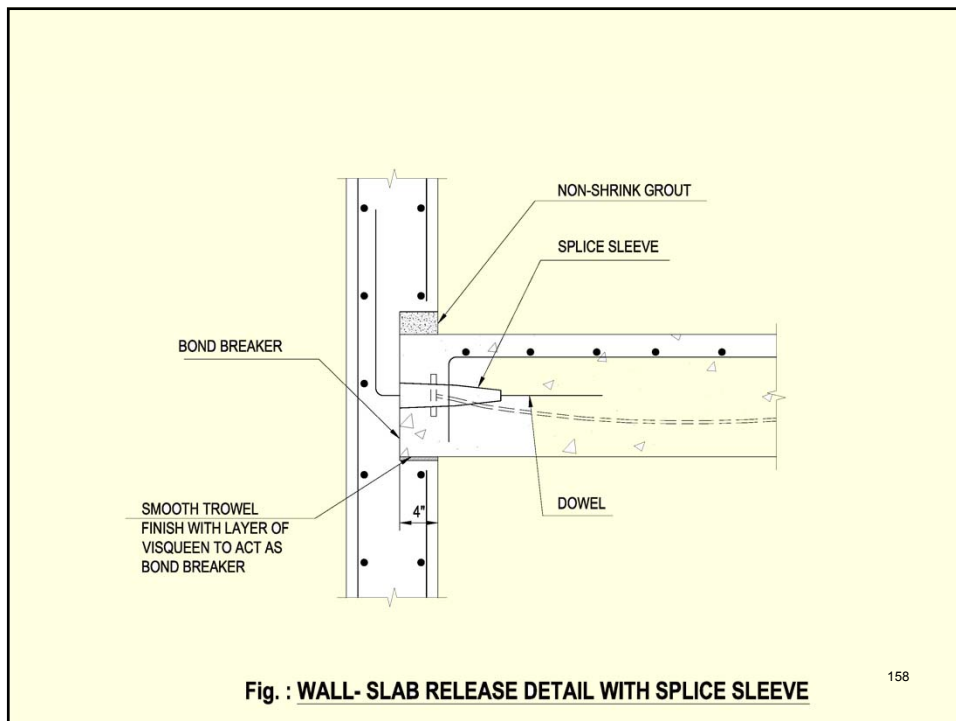
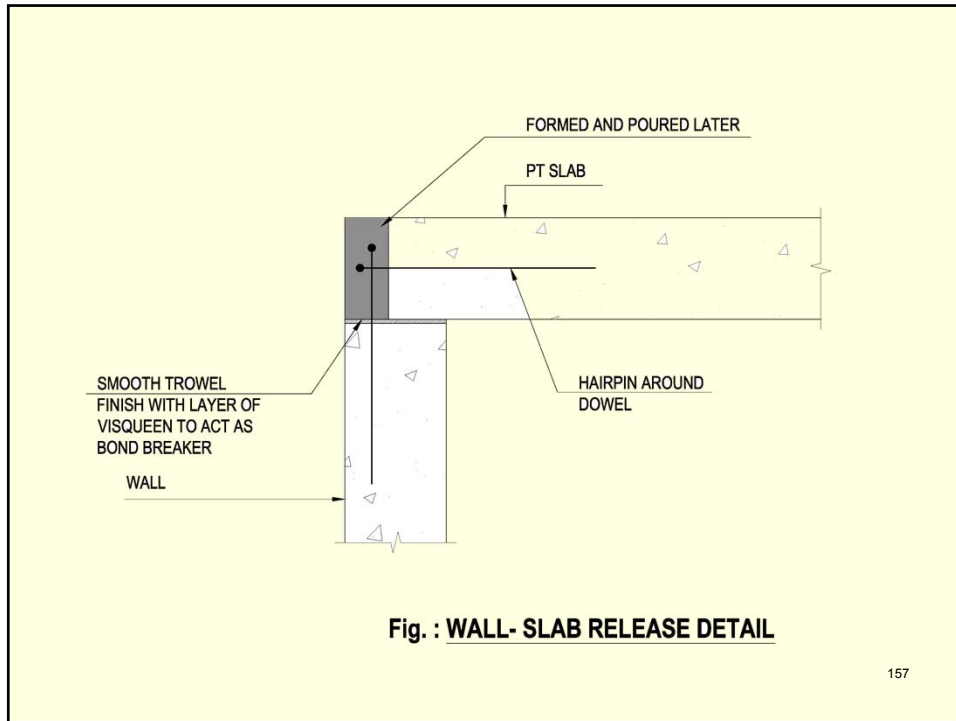
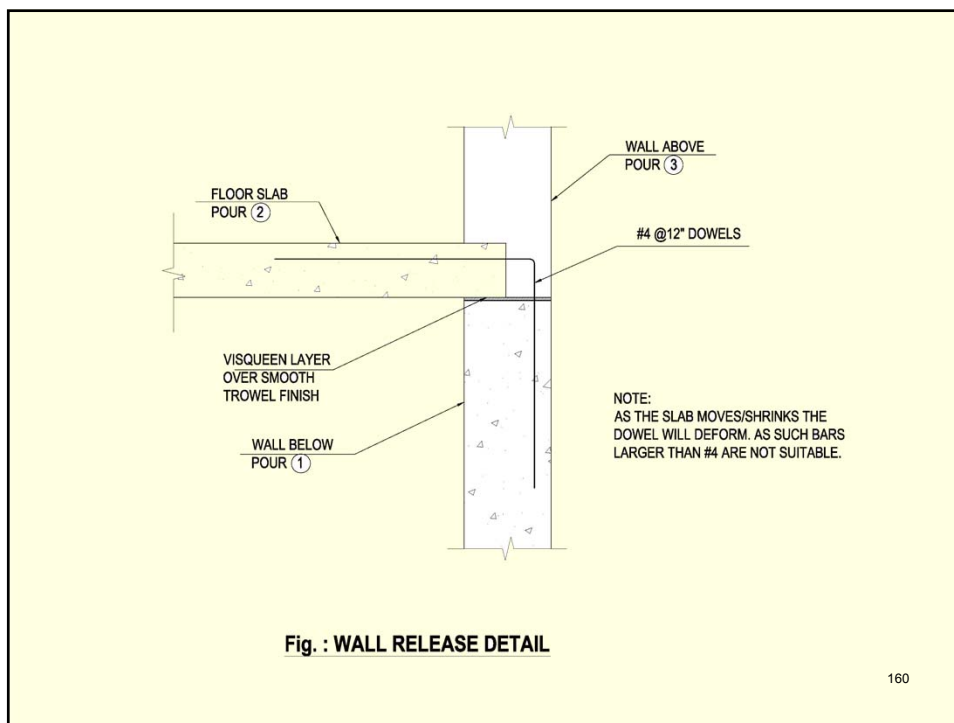
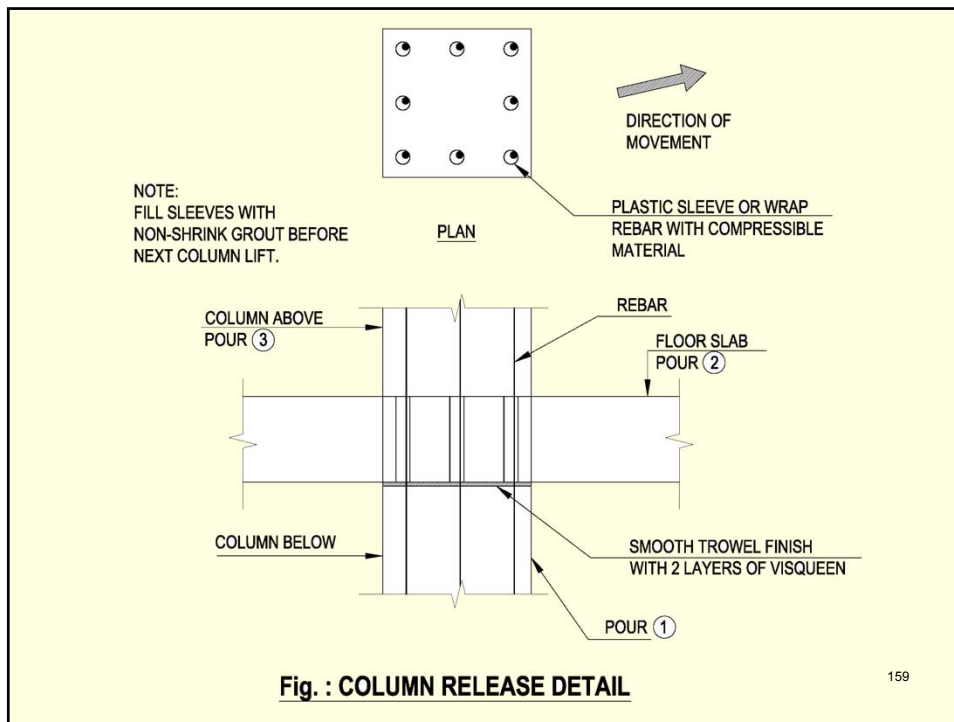


Fig. : WALL- SLAB RELEASE DETAIL

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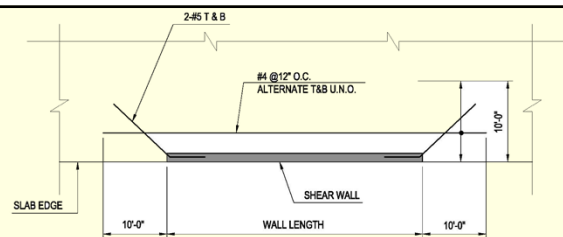


POUR STRIPS & CONSTRUCTION JOINTS

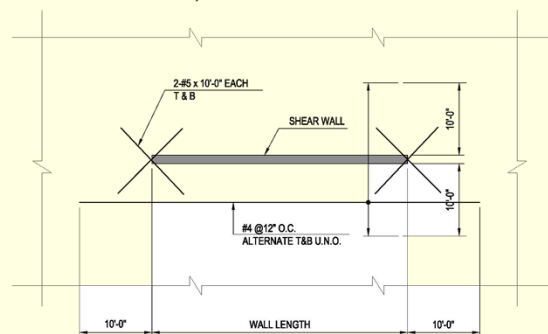
E] ADDITION & IMPROVED LAYOUT OF MILD STEEL

- Additional mild steel placed at potential distress location such as non-released walls away from the center of the structure
- Steel placed parallel to the rigidly connected wall for a distance equal to $1/3^{\text{rd}}$ the transverse span
- Steel Quantity: $A_s = .0015A_c$
 $= 0.0015 \times 12 \times 8 = 0.144 \text{ in}^2 \rightarrow \#4 @ 16 \frac{1}{2}''$
 Steel placed alternatively at top and bottom of slab.
- Not a code requirement but helps in controlling restraint cracks

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a.) PLAN AT EXTERIOR WALLS



b.) PLAN AT INTERIOR WALLS

Fig. : ADDITIONAL REINFORCEMENT AT SHEAR WALLS

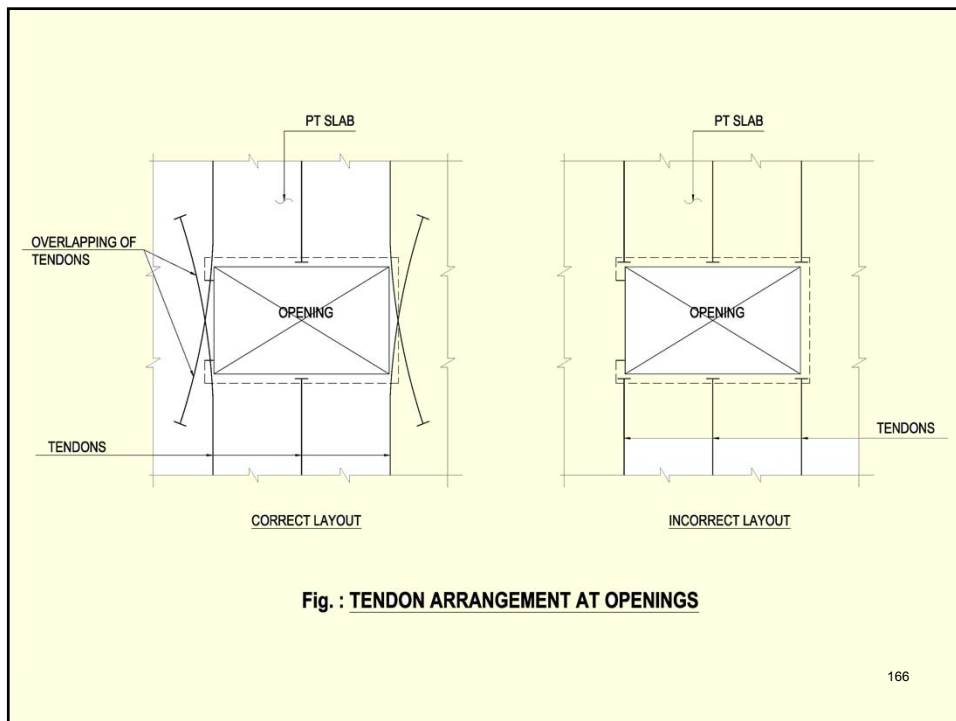
164

POUR STRIPS & CONSTRUCTION JOINTS

F] ADDITION & IMPROVED LAYOUT OF TENDONS

- Provide additional tendons in areas with heavy losses.
- This can be friction loss or loss of PT force due to dissipation into walls or columns
- Criss-cross PT tendons at opening location. This helps in controlling cracks at corners of the opening.

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RESTRAINT CRACKS - POINTS TO CONSIDER

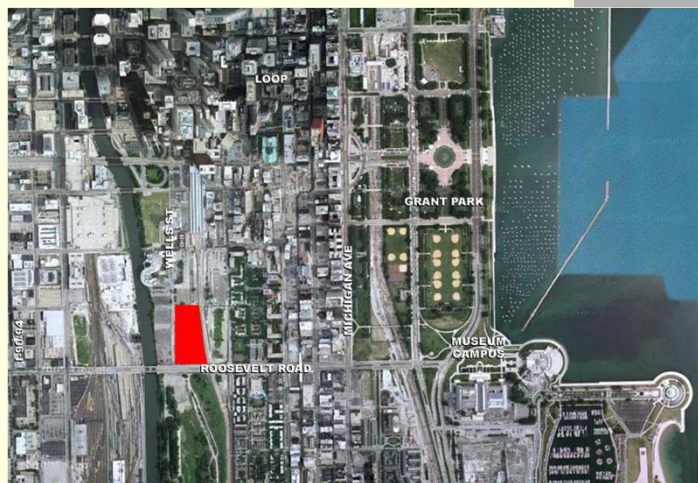
- Shortening cracks are frequent in post-tensioned slabs supported on walls and stiff columns
- Shortening cracks do not normally impair the structural integrity of the slabs.
- Impairment- Exposure of rebar and PT to corrosive elements, leakage and aesthetics.
- For PT slabs use various crack mitigation details to bring down the hypothetical displacement to 0.25"
- For example the closure concrete at the pour strips shall be placed at a time when the remaining calculated displacement of the slab each side of the strip is 0.25"
- For slab with significant support restraint such as perimeter walls it may be necessary to conduct a one time maintenance routine to repair shortening cracks. This should be discussed with in advance with the owner

Crack Treatment

- No Movement expected or desired – Use epoxy injection
- Movement expected and cannot be avoided – Use flexible sealant

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CASE STUDY – MULTI-USE COMPLEX Location Map



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CASE STUDY – MULTI-USE COMPLEX
Plaza View



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CASE STUDY – MULTI-USE COMPLEX
3D View



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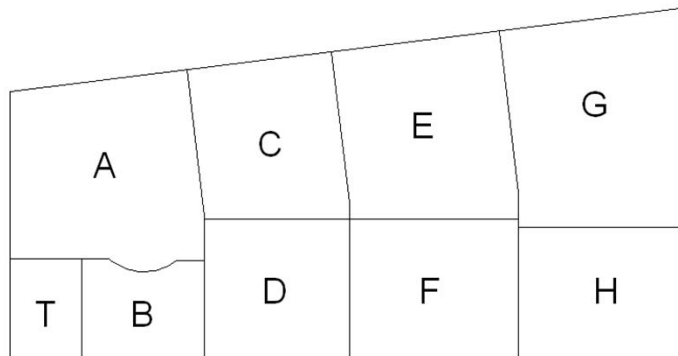
CASE STUDY – MULTI-USE COMPLEX Garage Floor Plate



950' x 500' Plate

171

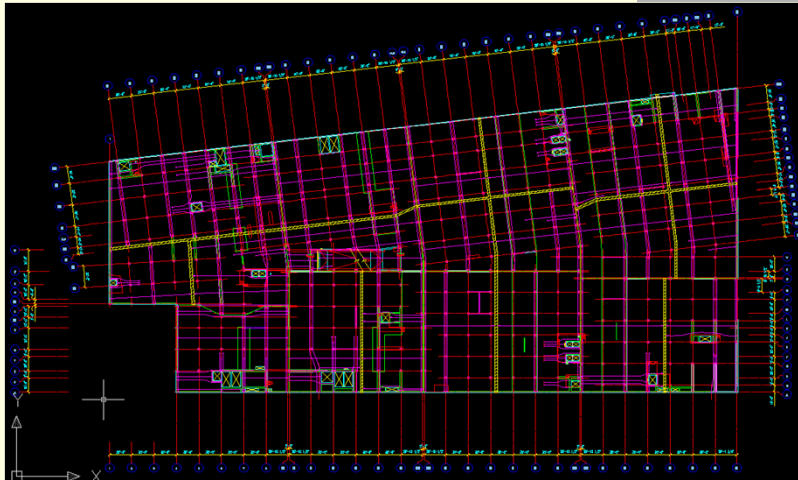
CASE STUDY – MULTI-USE COMPLEX Expansion Joints



Divided into 9 pieces

172

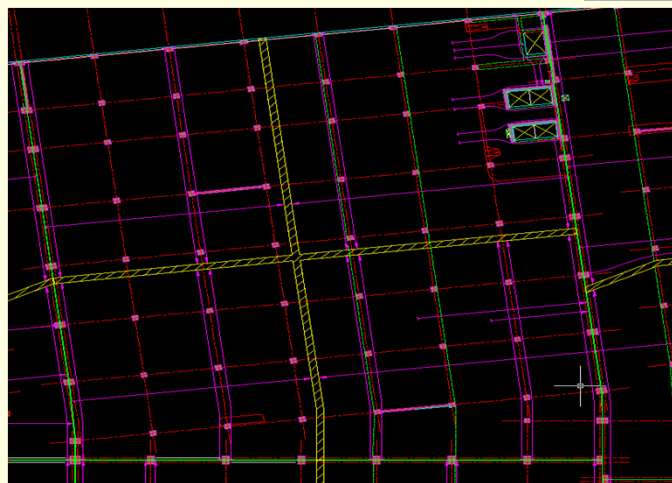
CASE STUDY – MULTI-USE COMPLEX Expansion Joints & Pour Strips



Divided into 9 pieces

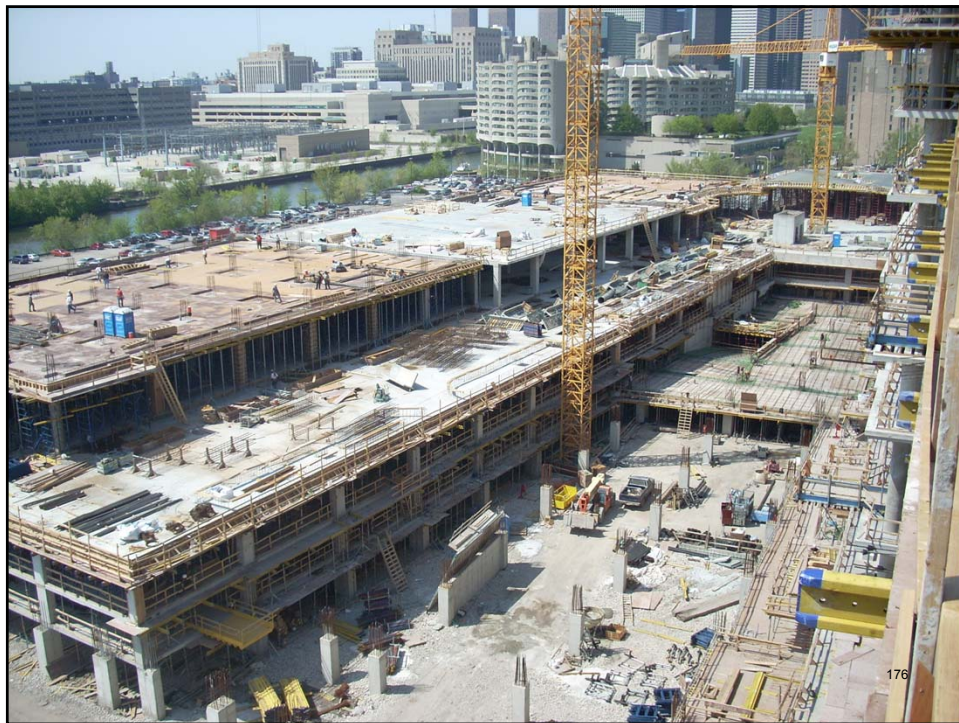
173

CASE STUDY – MULTI-USE COMPLEX Pour Strips – Area E



Pour Strips – 3' wide

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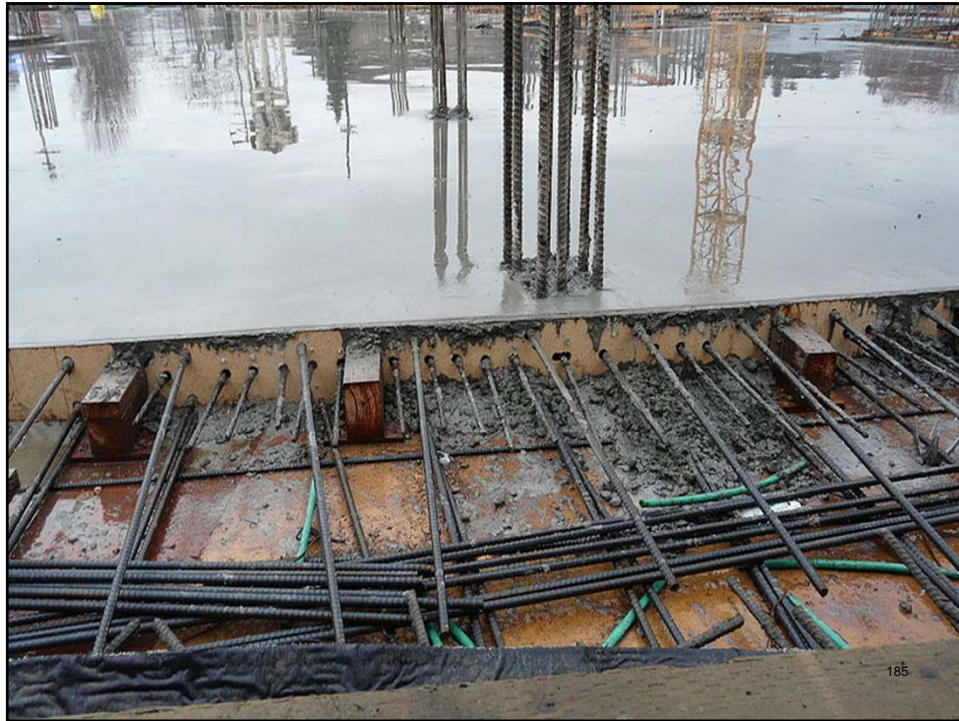












BARRIER CABLES WITH PT STRANDS

- Provided in Garages as a Guard Rail for Vehicles
- Why PT Cables Popular?
 - Provides openness
 - Good visibility
 - Economical
 - Good choice because of high tensile strength and low relaxation of steel
- IBC governs requirement for vehicle impact resistance
- Anchorages shall be capable of transmitting loads to structure
- End Supports (walls or columns) shall be designed for loads from cable

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BARRIER CABLES – IBC Provisions

Vehicular Protection

For typical garage barrier:

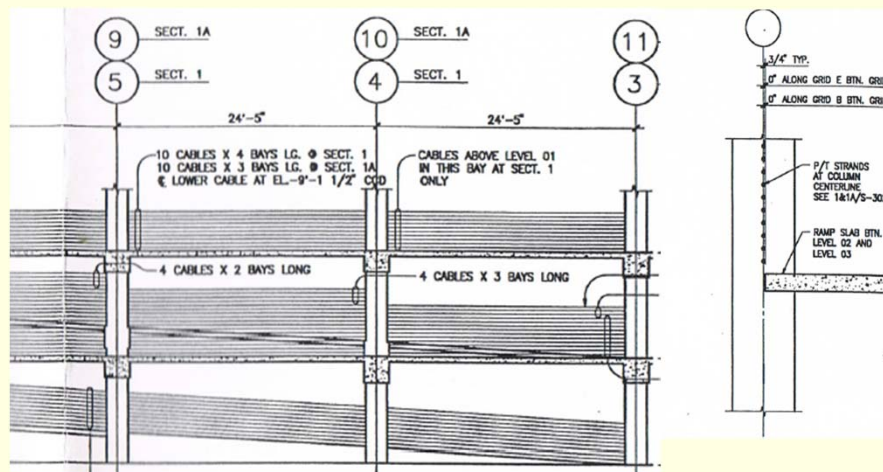
- ❖ P = 6000 lbs applied at 18" from floor horizontally in any direction
- ❖ Load applied to an area of 1 sq. ft (max)
- ❖ Barrier height = 24" minimum
- ❖ To be provided when elevation difference $\Delta = 12''$ between adjacent floors

Truck & Bus Protection shall follow an approved method:

- ❖ AASHTO guidelines – 10,000 lbs or more
- ❖ Delivery areas and Loading zones
- ❖ Traffic Patterns shall be considered including backing up

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BARRIER CABLES



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BARRIER CABLES – IBC Provisions

Pedestrian Protection

Fall Protection

Barriers shall be 42” high (min)
 4” sphere shall not pass thru any opening up to 34”
 8” sphere shall not pass thru any opening above 34”

Design Loads

50 plf applied in any direction
 200 lbs concentrated load in any direction on top of rail

Barrier Cables in Garage

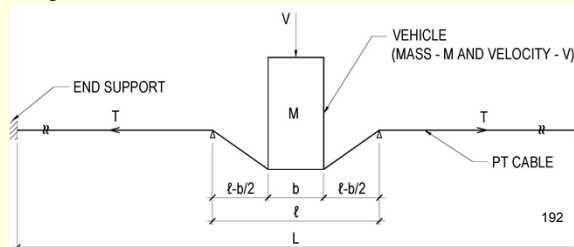
Since vehicular protection areas would also require pedestrian protection it is typical to provide barrier cables as follows:

11 cables spaced at 4” o.c with the first cable at 3.5” from floor
 Total height = 43.5”

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BARRIER CABLES – Failure Mechanisms

- Failure of Anchorage System
- Failure due to excessive deflection of the cable
- Breaking up of Cable on Impact
- Failure of support columns, walls
- Each of the above failure modes shall be checked in design of the barrier cables
- Lot of times the support columns and walls are not checked for adequacy and may cause problems



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BARRIER CABLES – Design

ENERGY METHOD

- Originally presented by Presswalla – Rational approach
- Kinetic energy of moving vehicle is converted to cable force (T) and deflection (d)
- The barrier cable of total length L is imparted an initial tension Fe
- The total length may have a number of spans but the design span (l) is the one with the longest span
- The tension in the cable when hit by a vehicle of mass, M moving at velocity V and resisted by N cables is given by:

$$T = [(AE/L)(MV^2/N) + Fe^2]^{1/2}$$

- Deflection of the cable due to impact of the vehicle is then calculated by:

$$d = [\{ (T-Fe)L / (2AE) + 1 - b \} * (T-Fe)L / (2AE)]^{1/2}$$

- If $d > 18''$ (allowable deflection) then some revisions are required:
 - 1] Choose a higher value of Fe or
 - 2] Add anchors on intermediate columns to reduce the value of L

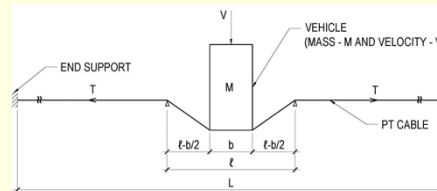
Both the above revisions may be required to bring the deflection and tension in the cable under control. In the above equations A & E are the area and modulus of elasticity of the cable respectively

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BARRIER CABLES – Design

Design Example – 8

Total length L = 200 ft
 Number of equal spans = 10
 Span length = 20'-0"
 Min Ultimate Tensile Strength, MUTS = 270 ksi
 Yield Strength $F_y = 90\%$ of MUTS
 Weight of vehicle = 4000 lbs
 Width of vehicle, b = 5'
 Velocity, V = 6 mph = 8.8 ft/sec



Cable Area A = 0.153 in² → 7 wire strand
 Modulus of Elasticity, E = 28.5 x 10⁶

Selecting an initial tension force in the cable, Fe = 4000 lbs & assuming 3 cables effective (N=3)
 Note that this force should be such that when the cable is tensioned with this force there should be no noticeable sag.

Tension in the cable: $T = [(AE/L)(MV^2/N) + Fe^2]^{1/2}$
 $= [(0.153 * 28.5 * 10^6 / 200) * (4000 / 32.2 * 8.8^2 / 3) + 4000^2]^{1/2}$
 $= 9268 \text{ lbs}$

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BARRIER CABLES – Design

Design Example 8 – cont.

Deflection of the cable:

$$\begin{aligned}
 d &= [\{(T-F_e)L/(2AE) + 1 - b\} * (T-F_e)L/(2AE)]^{1/2} \\
 &= [\{(9268 - 4000) * 200 / 2 / 0.153 / 28.5 * 10^6\} + 20 - 5] * (9268 - 4000) * 200 / 2 / 0.153 / 28.5 * 10^6]^{1/2} \\
 &= [(0.121 + 15) * 0.121]^{1/2} \\
 &= 1.35' = 16.2'' \\
 &< 18''
 \end{aligned}$$

Since $d < 18''$ (allowable deflection) then the revision to initial tension force and/or addition of intermediate anchor points are not required.

Check IBC requirement of 6000 lbs force applied on the barrier cable force system.
This force induces a tensile strength of:

$$\begin{aligned}
 T &= P/N * 1 / 4d \\
 &= 6000 / 3 * 20 / 4 / 1.35 = 7407 \text{ lbs} < \text{Tension calculated above} \rightarrow \text{OK}
 \end{aligned}$$

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Barrier Cables – Design

Design Example 8 – cont.

The required jacking force is summation of the initial force in the cable plus additional force to account for the seating loss. The seating loss is dependent on the anchorage system provided by the supplier but typically ranges from $1/4''$ to $3/8''$. Conservatively using the $3/8''$ value:

$$\begin{aligned}
 \text{Jacking Force on the cables} = F^j_e &= F_e + \text{Seating Loss} * A * E / 12 / L \\
 &= 4000 + 0.375 * 0.153 * 28.5 * 10^6 / 12 / 200 \\
 &= 4000 + 681 \\
 &= 4681 \text{ lbs}
 \end{aligned}$$

FORCE ON SUPPORTS

In this example there are no intermediate anchors and as such all the force is on the end wall or columns
There are a total of 11 cables each of which is initially stressed to 4000 lbs.

However, upon impact the three cable resisting the vehicle are stressed to a tension of 9268 lbs
As such, the total force on the column:

$$F_{col} = (9268 * 3) + (4000 * 8) = 59,804 \text{ lbs} \rightarrow \text{large force}$$

The column/wall should be checked for this force.

The column axial force and moments shall be combined with the effects of this horizontal force and the interaction checked. Short stub columns may be susceptible to this force.

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EXTERNAL PRESTRESSING

WHAT IS EXTERNAL PRESTRESSING?

- PT tendons placed outside the physical cross-section of the member
- Sometimes tendons are placed in groove/slots cut into the existing member
- PT force transferred to the member through end anchorages & deviators
- Deviators introduce forces in the structure which counteract service loads
- Beneficial effect on members – load carrying capacity is increased
- No bond between the external tendon and the existing structure
- Bonded or unbonded tendons may be used but the behavior of the assembly is that of unbonded tendons since the elongation of the tendon is independent of the member being reinforced

APPLICATIONS

- Construction of new bridges
- Rehab of existing bridges
- Strengthening existing building components
 - Mostly concrete components strengthened
 - Retrofit of steel, timber and other framing also done

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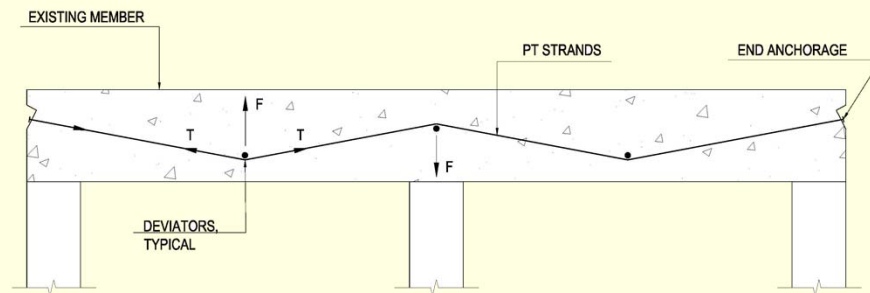
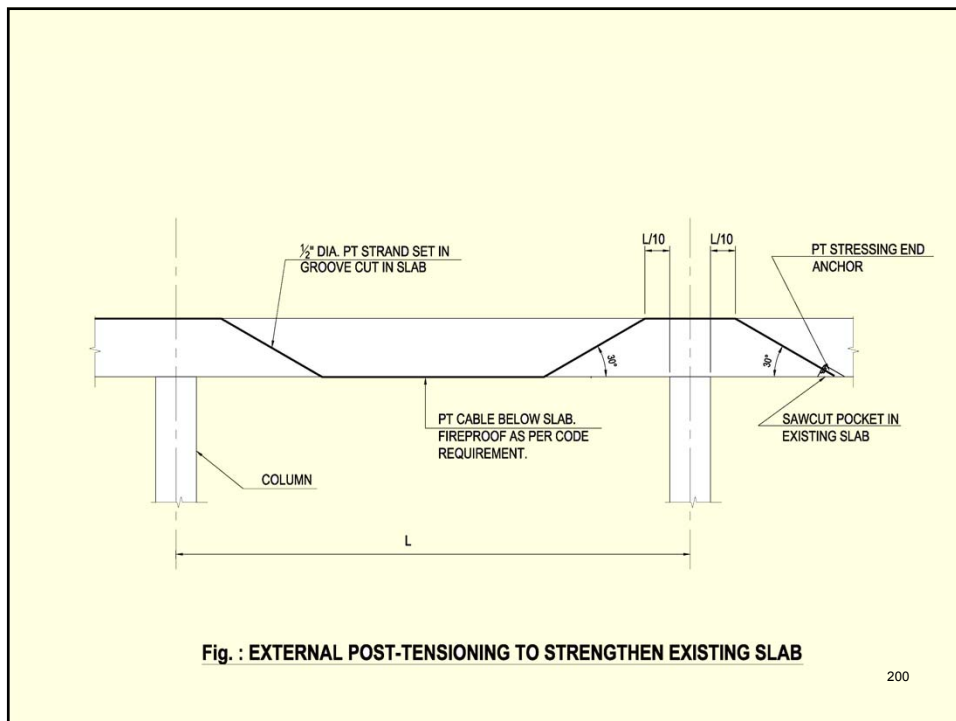
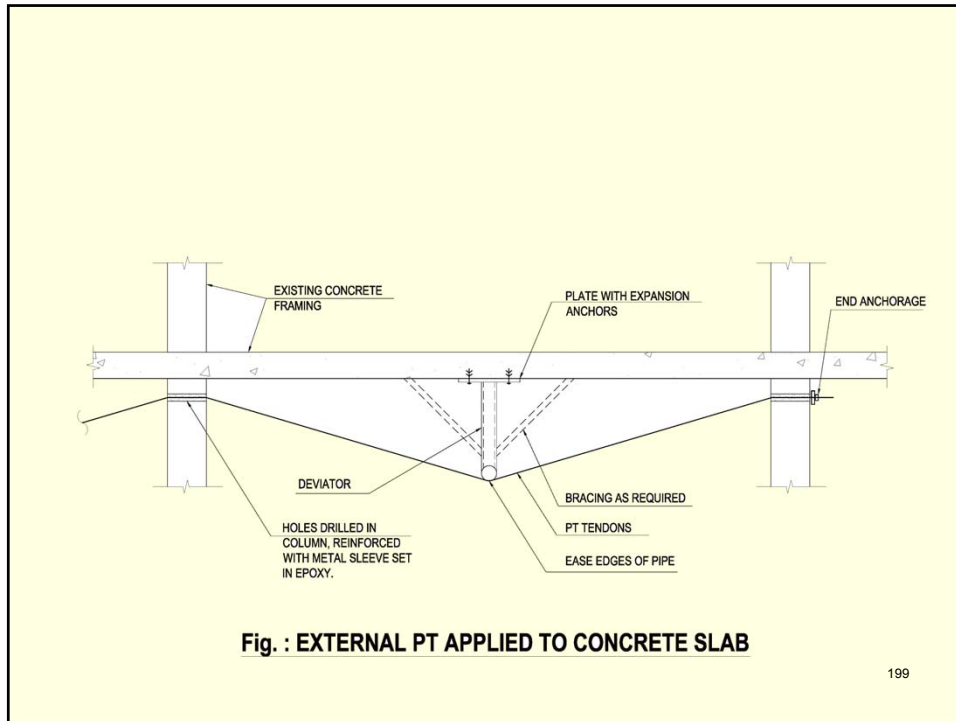


Fig. : EXTERNAL POST-TENSIONING

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EXTERNAL PRESTRESSING

MATERIALS

- Typically 0.5" and 0.6" dia. strands conforming to ASTM A416
- Deviators – Structural steel saddles bolted to sides or bottom of member
 - Solid bar or grouted pipe protruding through the beam web
- Anchorage Assembly – Anchor plate with head and wedge assembly
 - Angle Bracket with stiffeners is commonly used and is attached with thru bolts into existing member
- Application of tension force to the strand
 - By Hydraulic Jacks
 - By use of Turnbuckles
 - By turning nuts on threaded rods

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EXTERNAL PRESTRESSING

ADVANTAGES

- Strengthening of existing structures can be easily done
- Structures can be of any material
- Ease of inspection and monitoring
- Ability to replace strands if needed
- Ease of concrete placement if some or all tendons are removed from the web of beams and placed externally (avoids congestion)
- Reduction of friction losses because of reduction of wobble in the tendons
- Beam webs can be thinner with external prestressing → Lighter structure
- Lighter precast members – Cheaper transportation costs
- Lighter structure – Economical structure; less seismic forces

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EXTERNAL PRESTRESSING

DIS-ADVANTAGES

- Need access to the sides and ends of members for installation
- Need to make sure that columns and other supporting members can handle the extra forces introduced into the structure; they may need reinforcement too
- Overloading of structure due to external PT may cause unwanted cracks
- Needs corrosion protection in exterior and aggressive environments
- Needs fire protection
- Vulnerability to vandalism

Some of the above drawbacks are resolved by encasing the tendons in concrete cover by conventional methods or by shotcreting.

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EXTERNAL PRESTRESSING

DESIGN

- The uplift introduced in the structure can be easily calculated if the geometry of the structure is known.
- The deviators (also know as deflection blocks) should be designed for the uplift force
- The end anchorage introduces new forces into the structure.
 - The anchorage shall be designed for the factored force in the tendons
 - The externally prestressed member should also be checked for these forces
- If analysis requires additional bonded reinforcement then C-shaped channels can be bolted to the underside of the slab
- The ends of the member being reinforced may not be accessible and in this case the load will have to be transferred through side or bottom mounted bracket which may introduce eccentric forces into the structure.

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DRISCOLL BRIDGE FLOOR BEAM STRENGTHENING-NJ



Slide courtesy of DSI - America

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EXTERNAL PRESTRESSING

HISTORY

German engineer Franz Dischinger patented the use of External tendon in PT
 First used in design of bridge at Aue in Saxony in 1936

CONCLUDING REMARK

- The external prestressing method is a practical, economical and expedient method for:
 - repair or strengthening of existing structural members in buildings and
 - in rehabilitation of bridges and
 - reducing web thickness of beams in new precast construction thereby effecting savings in materials and transportation costs.

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APPLICATION OF PT TO BRIDGE STRUCTURES

COMMON TYPE

- GIRDER BRIDGES
 - I – Beams, Bulb Tees
 - Single Cell box section
 - Multiple Cell box section
 - T - Shapes

BENEFITS OF PRE-STRESSING IN BRIDGE DESIGN

- Cost effective
- Lighter & Slender than Reinforced Concrete
- Works for Curved Highway Alignments
- Durability
- Fast Construction
- Aesthetic Expression

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APPLICATION TO BRIDGE STRUCTURES

GIRDER BRIDGES

- ❑ By far the most popular and common type of bridge structure
- ❑ Spans range from 75' to 750' – Large spans are of the Hollow cell type
- ❑ Typically the span to depth ratio is of the order of 20:1
- ❑ Typically bonded tendons used in Bridge construction
- ❑ I beam – Bulb Tees
 - AASHTO Girders
 - PCI Girders
 - These type of girders are usually precast & pre-tensioned in the yard and brought to the site for installation

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APPLICATION TO BRIDGE STRUCTURES

- ❑ SINGLE CELL BOX GIRDER – efficient and versatile cross-section
 - The section consists of a top slab , two slightly inclined webs and a bottom slab
 - The bottom slab is usually 6-8" thick while the web is from 9-12" thick
 - The top slab thickness depends on the spacing between the web and could range from 8-12" thick
 - The section is strong in bending and stiff in shear and torsion.

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Pre-Cast Segments

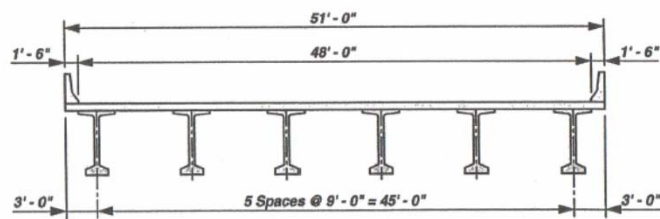


Ramp over I 95, Florida

Slide courtesy of DSI - America

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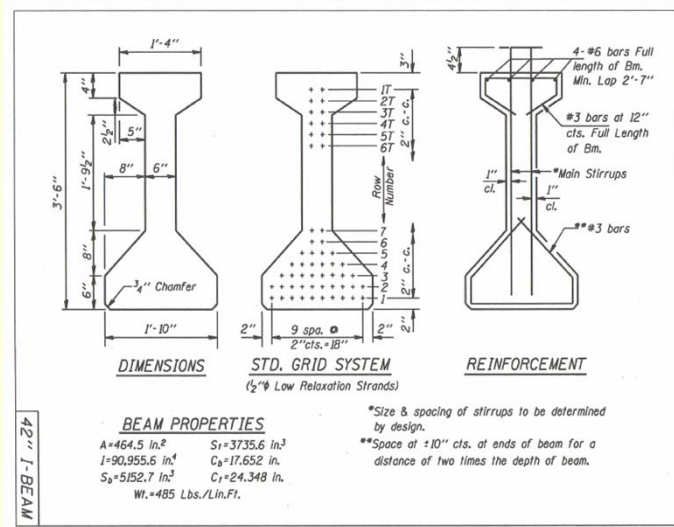
GIRDER BRIDGE



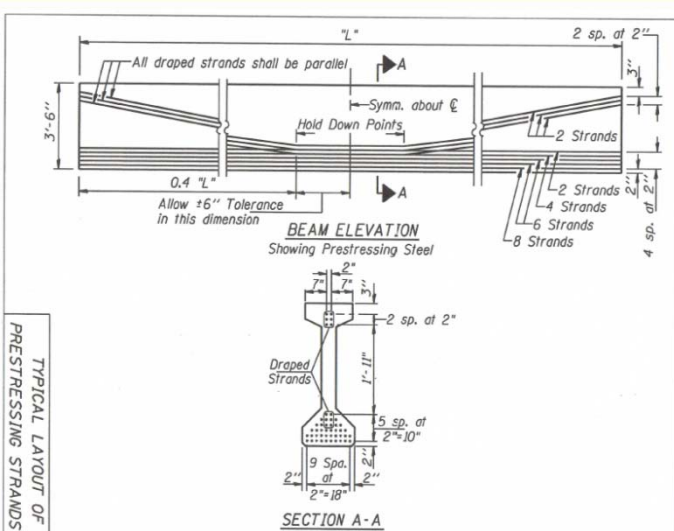
Cross Section of Bridge With Six PCI BT-72 Bulb Tee Girders at 9'-0" Spacing

212

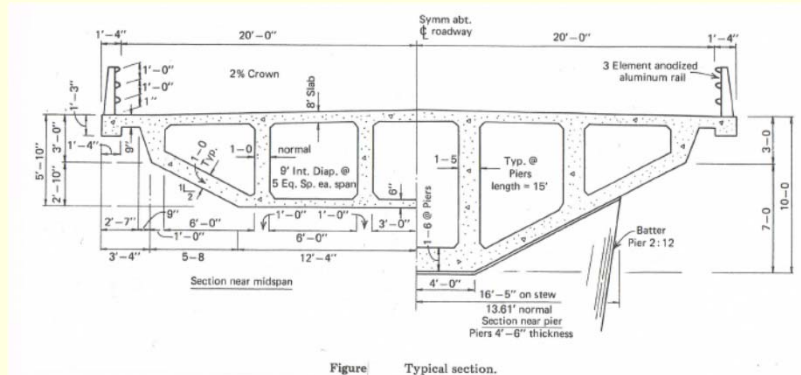
GIRDER BRIDGE



GIRDER BRIDGE



Multi-Cell Box Girder Bridge

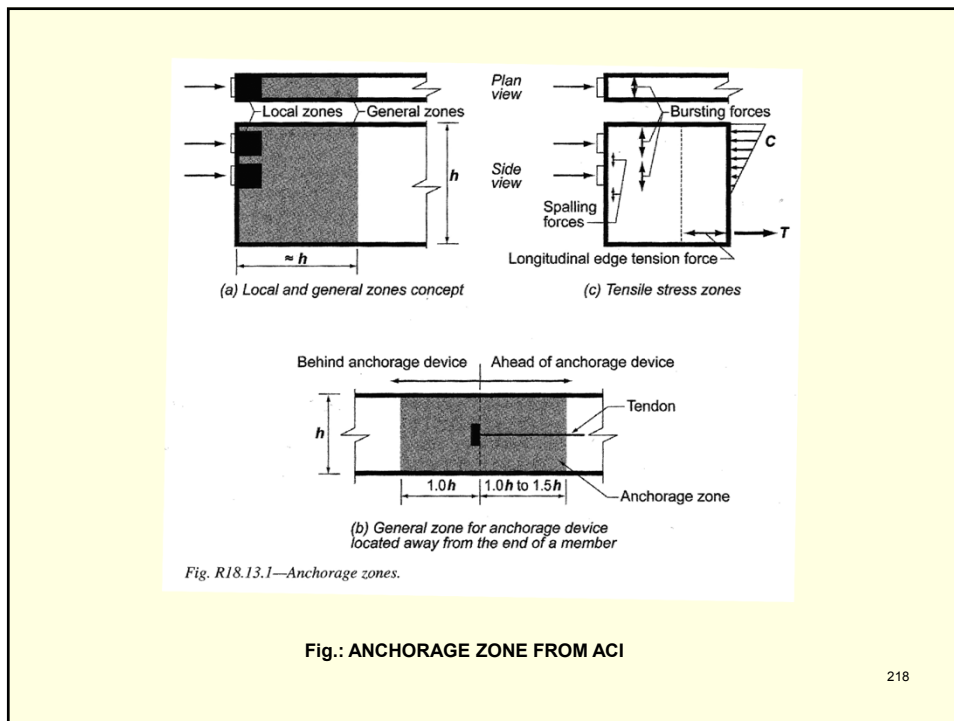
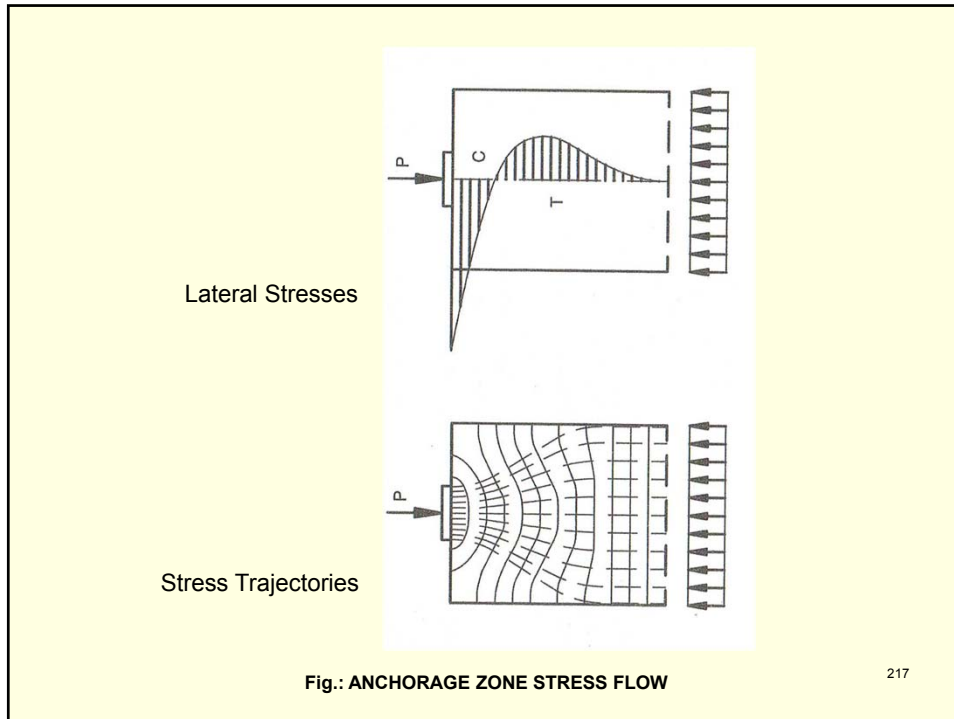


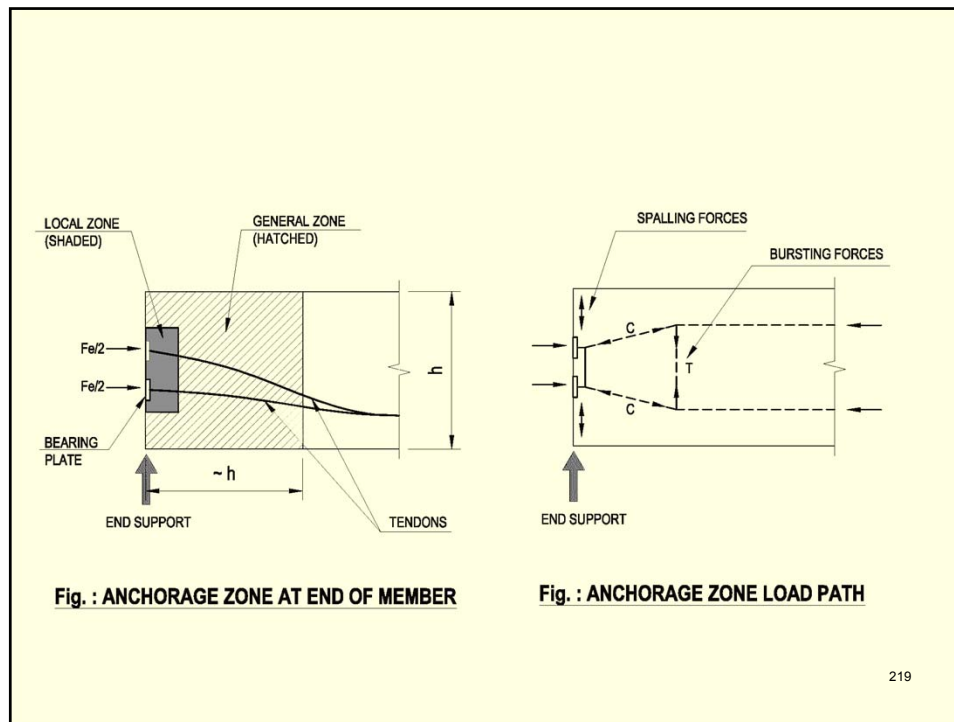
215

ANCHORAGE ZONES IN POST-TENSIONED CONCRETE

- ❑ Anchorage Zone – In post-tensioned members, this is the portion of the member through which the concentrated pre-stressing force is transferred to the concrete and distributed more uniformly across the section. For beams, the extent of the Anchorage Zone is equal to the largest dimension of the cross section for the beam and for slabs it is equal to the thickness of the slab. The depth and width of the anchorage zone are equal to those of the member.
- ❑ Anchorage Device – The hardware used for transferring a post-tensioning force from the prestressing steel to the concrete.
- ❑ Anchorage zone is considered to consist of two zones: (see figure on next page)
 - ❑ Local Zone – The rectangular prism of concrete immediately surrounding the anchorage device and any confining reinforcement
 - ❑ General Zone is the total anchorage zone as stated in the Anchorage Zone definition above and includes the local zone

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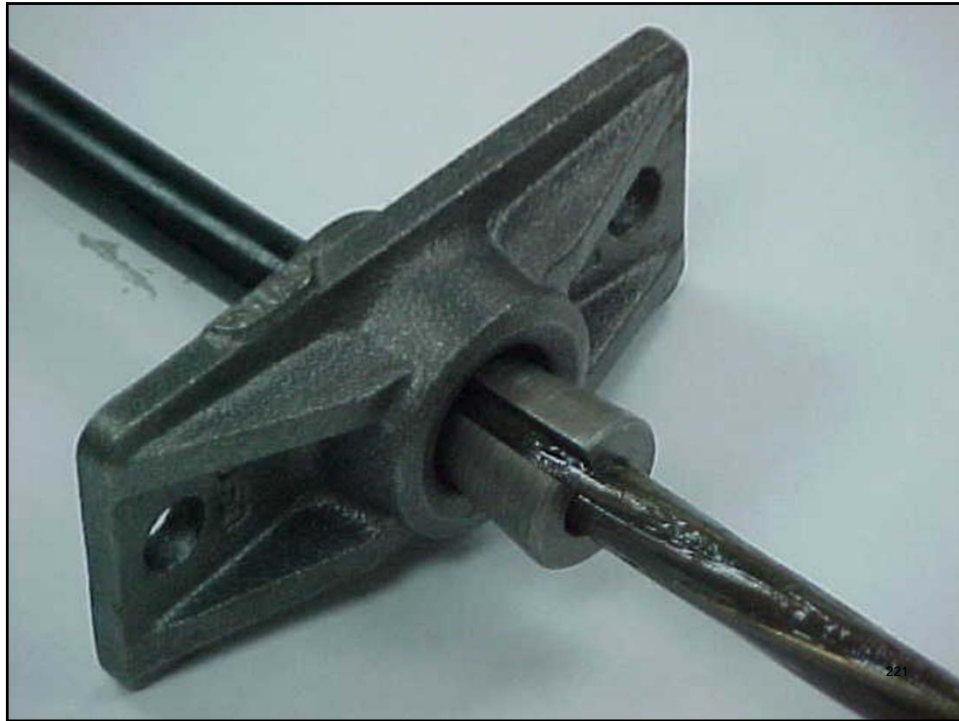




ANCHORAGE ZONES IN POST-TENSIONED CONCRETE

LOCAL ZONE

- ❑ In the Local zone, the concrete compressive stresses may exceed the acceptable values for unconfined concrete and special reinforcement may be required.
- ❑ As per PTI, the PT system supplier is responsible for the design and testing of the tendon anchorage components. This includes:
 - Proper performance of bearing plates
 - Local zone confinement reinforcement which is system dependent.
- ❑ Two types of bearing plates are used:
 - Basic bearing plates which are designed using a rational design approach based on design limits on bearing pressure, strength and stiffness requirements and no testing is required. These plates are somewhat similar with all PT system suppliers
 - Special bearing plates for which load transfer test record/data are required. These bearing plates are proprietary and normally different across the PT system suppliers.

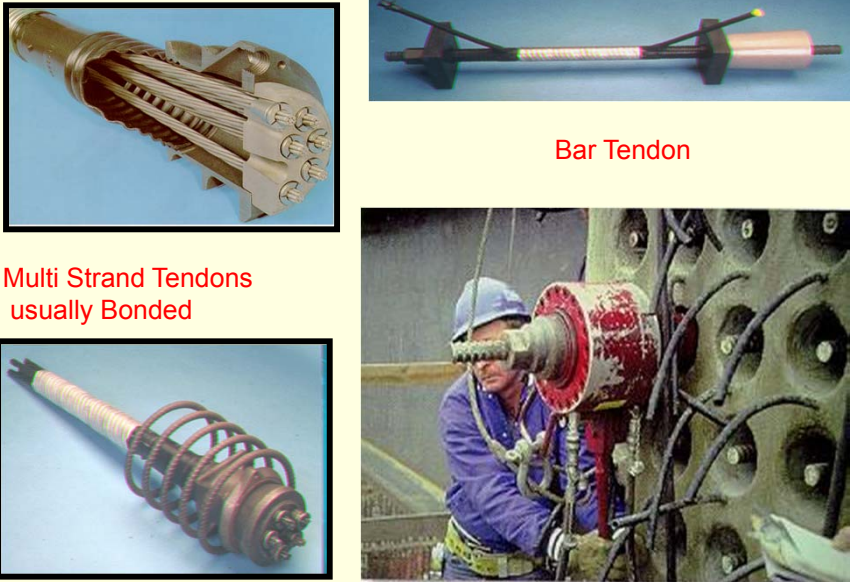


MULTI-STRAND STRESSING



Slide courtesy of DSI - America

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Bar Tendon

Multi Strand Tendons usually Bonded

Slide courtesy of DSI - America 223

ANCHORAGE ZONES IN POST-TENSIONED CONCRETE

LOCAL ZONE – contd.

- ❑ The design of **local zone** shall be based upon the factored prestressing force, P_{pu} calculated as follows:

$$P_{pu} = (1.2)(0.94) f_{py} * A_{ps} \leq (1.2)(0.8) f_{pu} * A_{ps} \quad [\text{ACI R18.13.2}]$$

where: f_{py} = Yield strength of PT steel
 f_{pu} = Tensile strength of PT steel
 A_{ps} = Area of PT steel
 1.2 is the load factor

Note that the Φ value (strength reduction factor) used for bearing plate and the confinement reinforcement is 0.85 as per ACI 9.3.2.5.

- ❑ AASHTO gives detailed definition of the local zone dimensions. For isolated bearing plates the depth of the local zone = $b + 2c$ where “b” is the width of the bearing plate and “c” is the min cover required each side of the bearing plate

ANCHORAGE ZONES IN POST-TENSIONED CONCRETE

GENERAL ZONE

- ❑ The design of the General zone shall be based upon the factored prestressing force, P_{pu} shown under Local zone discussion
- ❑ Reinforcement shall be provided where required to resist bursting, spalling and longitudinal edge tension forces induced by the anchorage device in the General zone.
- ❑ For intermediate location of anchorages, the General zone extends not only over the portion of the structure ahead of the bearing plate but also for a similar length behind the bearing plate.
- ❑ As per PTI, the design of the General zone (other than the local zone area) is the responsibility of the Design Engineer, because the behavior of the General zone depends primarily on tendon forces, tendon arrangement, stressing sequence and geometry of the structure.

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ANCHORAGE ZONES IN POST-TENSIONED CONCRETE

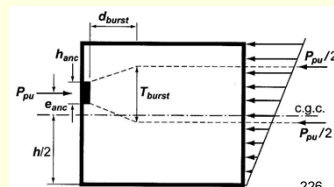
GENERAL ZONE – contd.

- ❑ The General zone design is usually based on one of the following methods:
 - ❑ Finite element method
 - ❑ Strut & tie models
 - ❑ Simplified equations if applicable. These methods are not be used in a number of situations including non-rectangular sections, discontinuities in General Zone and where edge distances to the anchors do not satisfy the min. requirements - See ACI[18.13.5.2]
- ❑ A simplified method proposed in ACI commentary is the Guyon's method which can be used to calculate the magnitude and location of the bursting stress in the General zone.

$$T_{burst} = 0.25 \Sigma P_{pu} (1 - h_{anc} / h) \quad [ACI R18-1]$$

$$d_{burst} = 0.5 (h - 2 * e_{anc}) \quad [ACI R18-2]$$

For definitions of the variables used in the above equations see figure.



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ANCHORAGE ZONES IN POST-TENSIONED CONCRETE

GENERAL ZONE – contd.

- ❑ Min. Requirements for anchorages of mono-strand tendons 0.5" dia. or smaller in normal-weight concrete slabs

Mono-strand means each tendon having its own bearing plate as opposed to multi-strand tendons

As per ACI [18.14.2]

1. Two horizontal bars (#4 min) shall be provided parallel to the slab edge and they shall extend at least 6" either side of the outer edges of the anchors. These shall be placed in front of the anchors but within a distance of $h/2$ from the anchors (h = slab thickness)
2. For each group of six or more anchors, a hairpin bar or closed stirrup shall be provided between each anchor and on the outer side of the anchors. If the anchors are spaced more than 12" apart then these hairpin bars are not required. See figure for the placement of these bars.
3. The above requirement can be waived if a detailed analysis (finite element or equivalent) shows such reinforcement is not required.

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POST-TENSIONING SYSTEMS

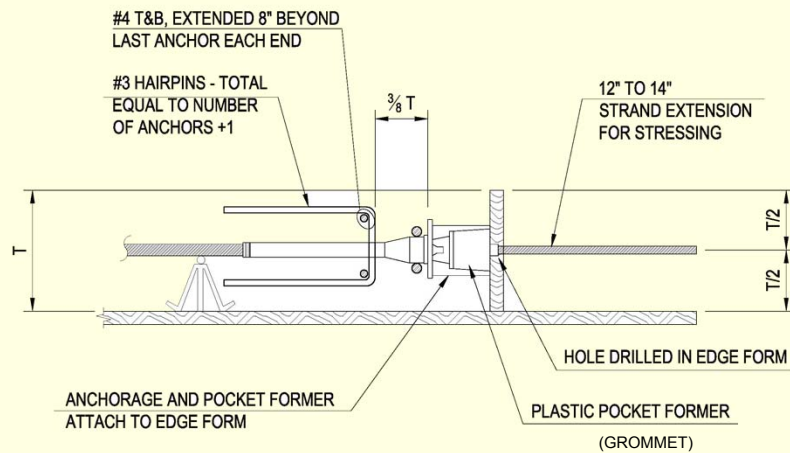
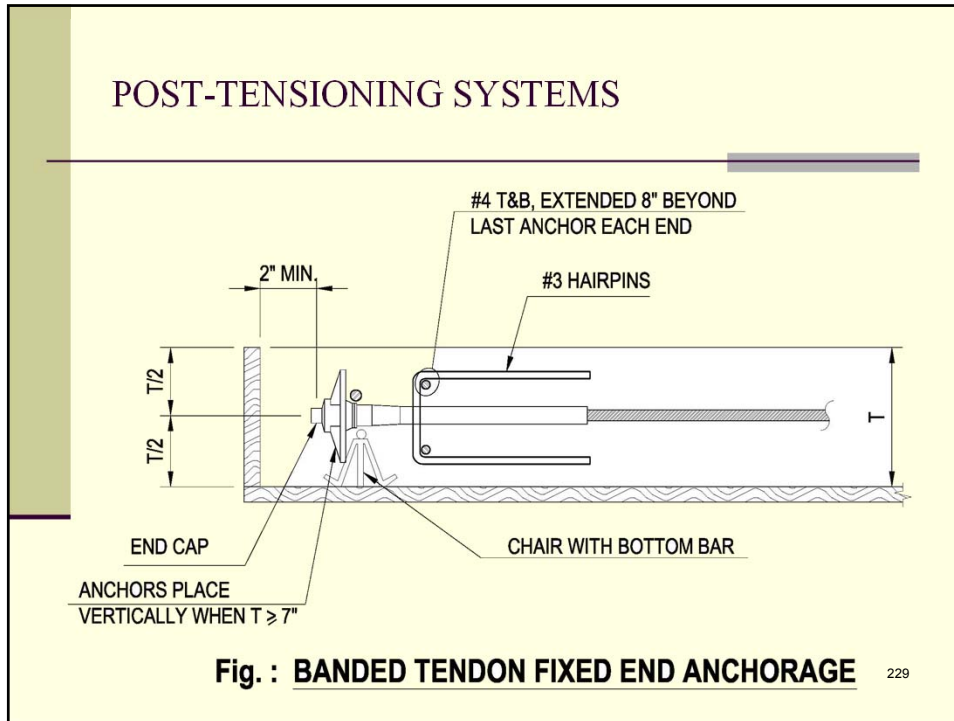


Fig. : BANDED TENDON STRESSING END ANCHORAGE

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ANCHORAGE ZONES IN POST-TENSIONED CONCRETE

GENERAL ZONE – contd.

- ❑ Requirements for anchorages of mono-strand tendons larger than 0.5" dia.
 Also, smaller dia. tendons in light-weight concrete will fall into this category
 As per ACI [18.14.2.4]
 Minimum reinforcement shall be based upon a detailed analysis satisfying ACI [18.13.5]
- ❑ Design for groups of mono-strand tendons in beams and girders
 Design of General Zones for groups of monostrand tendons in beams and girders shall meet the requirements of ACI [18.13.3 through 18.13.5]
- ❑ Design of Anchorage zones for multi-strand tendons
 Design of General Zones for groups of multi-strand tendons shall meet the requirements of ACI [18.13.3 through 18.13.5]

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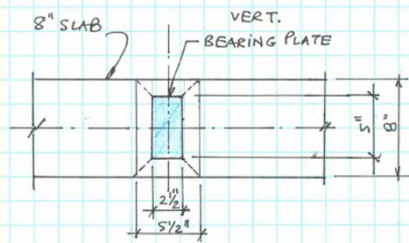
ANCHORAGE ZONE DESIGN

Example 9: Compute the bearing plate size for a 1/2" dia monostrand tendon.

Design Data

$f'_c = 4,000$ psi
 $f'_{ci} = 3,000$ psi
 $t = 8$ " slab
 $A_{ps} = 0.153$ in²
 $f_{pu} = 270,000$ psi (low relax)
 $f_{py} = 0.9 f_{pu}$

Assume 15 ksi total losses and a trial bearing plate size of 5" x 2 1/2" placed vertically in the 8" slab



As per PTI,

When no local zone confinement reinforcement is provided the allowable bearing stress under the max. allowable tendon jacking force (F_j) is limited to:

$$f_{cpi} = 0.5 * f'_{ci} * \sqrt{A/A_g} \leq 1.0 f'_{ci}$$

where: A_g = Gross bearing plate area = 2.5 * 5 = 12.5 in²
 A = Enlarged area geometrically similar to the bearing plate area = 8 * 5.5 = 44 in²
 $F_j = 0.8 * A_{ps} * f_{pu} = 0.8 * 0.153 * 270 = 33.0$ kips

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ANCHORAGE ZONE DESIGN

Example 9 – contd.

$$f_{cpi} = 0.5 * f'_{ci} * \sqrt{A/A_g} = 0.5 * 3 * \sqrt{44/12.5} = 2.81 \text{ ksi} \leq 1.0 f'_{ci} = 3 \text{ ksi}$$

Thus, $A_b = F_j / f_{cpi} = 33 / 2.81 = 11.7$ in²

Assuming that a 1" dia circular area is lost for bearing as the tendon passes through the plate, we have

$$A_g = A_b + \pi * d^2 / 4 = 11.7 + (3.14 * 1^2 / 4) = 12.49 \text{ in}^2$$

$$A_p = 5 * 2.5 = 12.5 \text{ in}^2 \rightarrow \text{OK}$$

Notes:

If the bearing plate is placed horizontally then we get a larger value of A which results in $f_{cpi} = 3$ ksi

$$A_b = 33 / 3 = 11 \text{ in}^2$$

$$A_g = A_b + \pi * d^2 / 4 = 11.0 + (3.14 * 1^2 / 4) = 11.78 \text{ in}^2 < A_p = 12.5 \text{ in}^2$$

The bearing plate size can be slightly reduced to 5" x 2 3/8" or 5 1/4" x 2 1/4"

If local zone reinf. is provided such that ($\rho > 0.02$) then the allowable bearing stress is increased as per:

$$f_{cpi} = 0.75 * f'_{ci} * \sqrt{A/A_g} \leq 1.5 f'_{ci}$$

This would result in a smaller bearing plate

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ANCHORAGE ZONE DESIGN

Example 9 – contd.

To complete checking the design of the bearing plate we should compute the bearing stresses under service load conditions:

$$f_{cp} = 0.6 * f_c * \sqrt{A/A_g} = 0.6 * 4 * \sqrt{44/12.5} = 4.5 \text{ ksi} > f_c = 4 \text{ ksi}; \text{ Use } f_{cp} = 4 \text{ ksi}$$

$$F_e = A_{ps} * (0.7 f_{pu} - 15) = 0.153 * 174 = 26.6 \text{ kips}$$

$$A_b = F_e / f_{cp} = 26.6 / 4.0 = 6.7 \text{ in}^2 \ll 11.7 \text{ in}^2$$

Normally this stage would not control for bearing plate design because of the larger value of f_c compared to f_{ci} and the reduced value of the prestress force after losses.

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ANCHORAGE ZONE DESIGN

Example 10: Compute the anchorage zone reinforcement for a banded tendon group of 8 tendons in a 8" PT slab.

Design Data

- $f_c = 5,000 \text{ psi}$
- $f_{ci} = 3,000 \text{ psi}$
- $h = 8" \text{ slab}$
- $A_{ps} = 0.153 \text{ in}^2$
- $f_{pu} = 270,000 \text{ psi (low relax)}$
- $f_{py} = 0.9 f_{pu}$

Fig. R18.13.5—Strut-and-tie model example.

Assume plate size of 5" x 2 1/2" placed horizontally ($h_{anc} = 2.5"$) at a spacing of 10" in the 8" slab

Using the simplified equations for calculating the bursting (tension) force in the slab:

$$T_{burst} = 0.25 \Sigma P_{pu} (1 - h_{anc} / h) \quad \text{[ACI R18-1]}$$

$$d_{burst} = 0.5 (h - 2 * e_{anc}) \rightarrow \text{Location of } T_{burst} \quad \text{[ACI R18-2]}$$

where: $\Sigma P_{pu} = (1.2) * (0.94) f_{py} * A_{ps} \leq (1.2) * (0.8) f_{pu} * A_{ps}$ Using $f_{py} = 0.9 f_{pu}$

$$= 1.01 f_{pu} * A_{ps} \leq \frac{0.96 f_{pu} * A_{ps}}{\text{controls}}$$

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ANCHORAGE ZONE DESIGN

Example 10 – contd.

$$T_{burst} = 0.25 \Sigma P_{pu} (1 - h_{anc} / h) \quad [ACI R18-1]$$

$$d_{burst} = 0.5 (h - 2 * e_{anc}) \rightarrow \text{Location of } T_{burst} \quad [ACI R18-2]$$

Thus, $\Sigma P_{pu} = 0.96 f_{pu} * A_{ps} = 0.96 * 270 * (8 * 0.153) = 317.3 \text{ kips}$

$$T_{burst} = 0.25 * 317.3 (1 - 2.5/8) = 54.5 \text{ kips} \rightarrow 54.5/6.67 = 8.17 \text{ kips/ft}$$

This force is spread over a distance of $10 * 8 = 80" = 6.67'$ in the slab

Reinforcement required to control the bursting force:

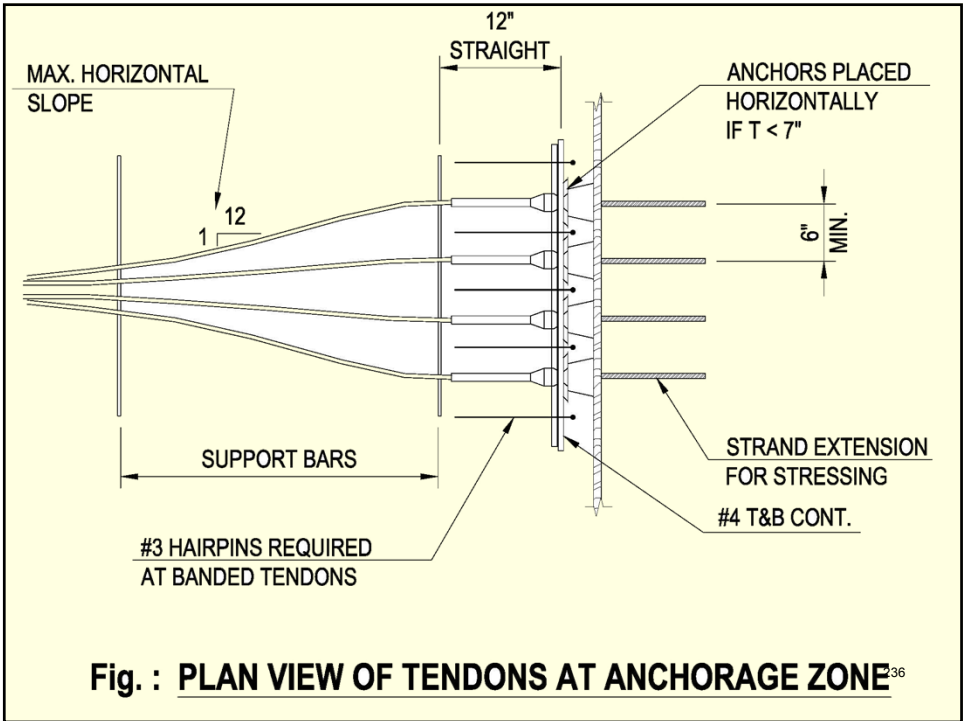
$$0.85 * A_s * f_y = T_{burst}$$

$$A_s = T_{burst} / (0.85 * f_y) = 8.17 / (0.85 * 60) = 0.16 \text{ in}^2 \rightarrow \#4@10" \text{ hair pin bar Ok}$$

This hair pin bar shall be placed in between the tendon anchors as shown in figure.

Location of vertical leg of the hair pin $d_{burst} = 0.5 (h - 2 * e_{anc}) = 0.5(8 - 0) = 4"$

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PT SLABS ON GROUND

Reference

- Design of Post-Tensioned Slabs-on-Ground, Third Edition, 2008
PTI (Post-Tensioning Institute)
- Research Work mostly done at Texas A & M University under the leadership of Dr. Robert Lytton.

Usage of PT

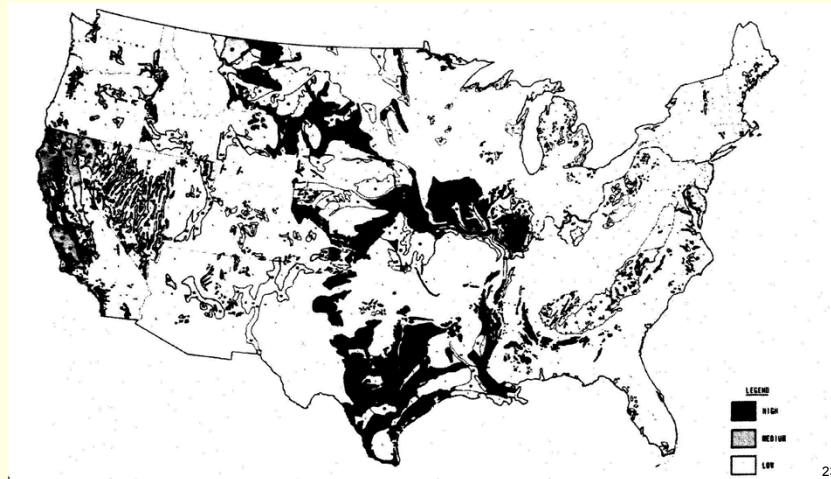
- 50% Structures (includes buildings & bridges)
- 50% Slab on Grade and foundations (includes tie-back anchors)
- Slab on Grade – Single largest application for PT tendons
- 72,000 tons of PT strands used in ground supported foundation slabs (2003) 237

PT SLABS ON GROUND

- ❑ Also called Floating Slabs
- ❑ Generally built on expansive soils or highly compressible soils where non-prestressed slabs have not performed satisfactorily.
- ❑ Where do we see most of PT slab-on-ground construction?
 - Texas – 50%
 - California – 25%
 - Rest – 25% (includes: LA, MO, NV, AZ, CO, FL, GA and others)
- ❑ See Map on next page

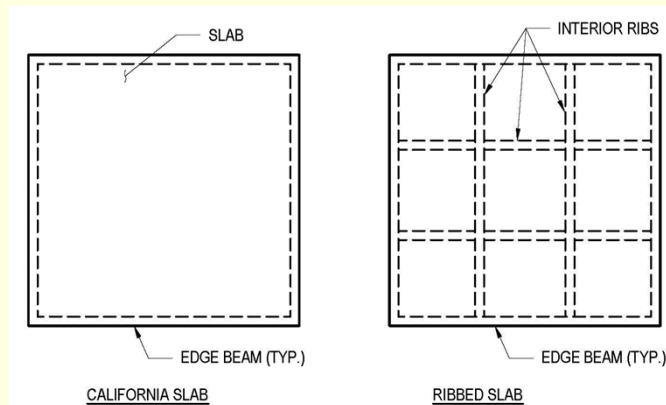
PT SLABS ON GROUND

EXPANSIVE SOIL MAP



PT SLABS ON GROUND - Types

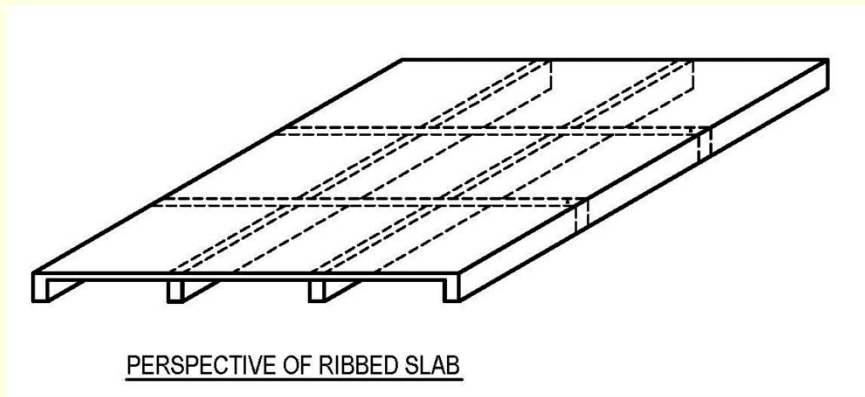
1. Ribbed Slab – Stiffening ribs at the bottom of the slabs are provided in both directions
2. California Slab – Uniform thickness slab with an edge rib at entire perimeter
3. Uniform Thickness Slab – No stiffening ribs; constant thickness slab



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PT SLABS ON GROUND - Ribbed Slab

Most common type



PERSPECTIVE OF RIBBED SLAB

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PT SLABS ON GROUND - Geotechnical Input

Geotechnical Input is of paramount importance because of complex effects of differential soil movements. Soil engineer shall classify site as expansive, or compressible or inactive for movements and then provide applicable soil parameter values

Edge moisture variation (e_m)

Distance beneath the edge of a slab or shallow foundation within which moisture will change due to wetting or drying influences around the perimeter of the foundation.

Edge Lift Case – The moisture in the soil is higher at the edges of the slab or foundation than in the center; also called edge swell, dishing or curling. This is a seasonal condition where the soil beneath the perimeter becomes wetter compared to the interior of the slab.

Center Lift Case – The moisture in the soil is higher at the center of the slab or foundation than at the edges; also called center heave or doming. Soil shrinks at the edges due to lower moisture content or when the moisture content of the slab in the interior increases and the soil expands because of this effect.

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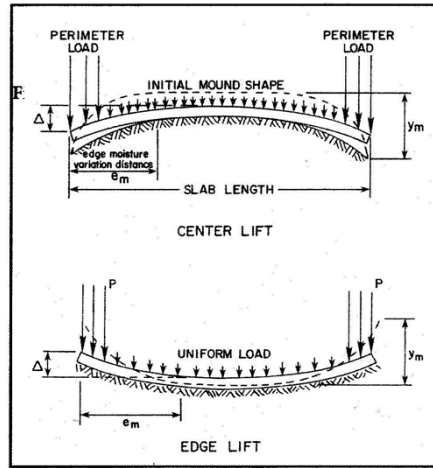
PT SLABS ON GROUND - Geotechnical Input

Differential Soil Movement (Y_m)

Also known as Differential Swell

This is the change in soil surface elevation at two locations separated by the e_m distance.

The Y_m value depends on a number of conditions including the type and amount of clay mineral, depth of clay layers, uniformity of clay layers, initial wetness, the velocity of moisture infiltration etc.



PT SLABS ON GROUND - Structural Parameters

❑ Shape Factor of Slab

- $SF = \frac{[\text{Foundation Perimeter}]^2}{\text{Foundation Area}}$
- If $SF > 24$, then some modifications to the foundation are required. This may include thickening of slab, adding ribs or doing soil treatment to reduce swelling among other things.

❑ Slab thickness

❑ Rib Spacing

❑ Rib Width and depth

PT SLABS ON GROUND - Minimum Prestress

Minimum prestress in slab

- To minimize joints and cracks in the Slabs on ground a minimum amount of prestress is used:

$$F_{e \text{ min}} = 0.05 * A$$

$$F_e = 26.6 \text{ k/tendon [include effect of losses and sub-grade friction on tendon]}$$

Assume another 5 k /tendon loss due to the effect of sub-grade friction

Then: $F_e' = 26.6 - 5 = 21.6 \text{ k/tendon}$

For a 12" strip of 6" thick slab

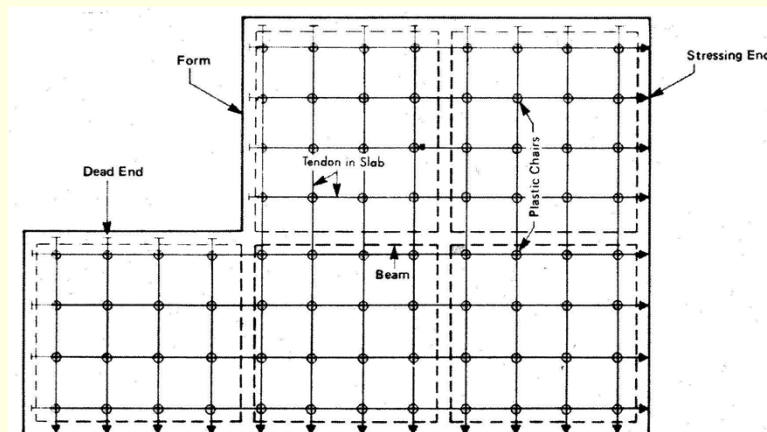
$$F_{e \text{ min}} = .05 * 12 * 6 = 3.6 \text{ k/ft}$$

$$\text{Spacing of tendons} = F_e' / F_{e \text{ min}} = 21.6 / 3.6 = 6' \text{ o.c.}$$

- Common practice is to have tendon spacing between 4' to 5' on centers based on calculations
- For highly expansive soils and compressible soils a higher value of prestress will result from calculations.

PT SLABS ON GROUND

TYPICAL TENDON LAYOUT



PT SLABS ON GROUND - Design Approach

Design Method for Ribbed Slabs

- Design assumes loss of support of foundation. The PT slab is designed to span this loss of support distance while staying within prescribed stress and deflection limits.
- Based on a working stress method
- Trial section is calculated based on each of the two cases of e_m

Center Lift

$$h = \left[\frac{(y_m L)^{0.205} S^{1.059} P^{0.523} (e_m)^{1.296} C_\Delta}{4560(z)} \right]^{0.824} \quad \text{where: } h = \text{slab thickness}$$

Edge Lift

$$h = \left[\frac{L^{0.35} S^{0.86} (e_m)^{0.74} (y_m)^{0.76} C_\Delta}{191(P)^{0.01}(z)} \right]^{1.176}$$

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PT SLABS ON GROUND - Design Approach (contd.)

where:

- L = Length of foundation
- S = Rib spacing
- P = Unfactored service line load acting along the perimeter not including self wt of the slab on grade
- C_Δ = Coefficient used to establish minimum foundation stiffness and is dependent on the superstructure construction material
- z = relative stiffness length

- Moments, shears under applied service loads and soil loading based on changes in climatic moisture are then calculated using equations. These equations were developed from empirical data and computer studies of a plate on elastic foundation. These equations are very complex/elaborate and take into account a number of parameters including e_m & Y_m and subgrade friction of the soil.
- Concrete stresses resulting from these moment & shears (uncracked section) are limited to specific allowable values:

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PT SLABS ON GROUND - Design Approach (contd.)

Allowable Concrete Tensile Stress: $f_t = 6 \sqrt{f_c}$
Allowable Concrete Compressive Stress: $f_c = 0.45 f_c$

- ❑ Differential deflection in the slab are then limited to acceptable values by providing minimum foundation thickness (stiffness).
- ❑ The above procedure is for design for design of ribbed foundation.
- ❑ To design a uniform thickness foundation the common approach is to design a ribbed foundation first and then convert to an equivalent uniform thickness foundation.

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PT Slabs on Ground

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FIELD ISSUES

PRIOR TO CONCRETE PLACEMENT

Do the PT Profiles match the control points given on the drawings?

- Measure HP and LP – should be within ACI & PTI tolerances
- Vertical Tolerance:
 - For $T < 8"$ → Tolerance = $\frac{1}{4}"$
 - For $8" < T < 24"$ → Tolerance = $\frac{3}{8}"$
 - For $T > 24"$ → Tolerance = $\frac{1}{2}"$ where: T = slab thickness
- See if the profile is a smooth parabolic profile without abrupt changes or reverse curvatures
- Check if LP are at midspans in the interior spans and slightly off in the exterior spans (0.4L to 0.45L from exterior support)

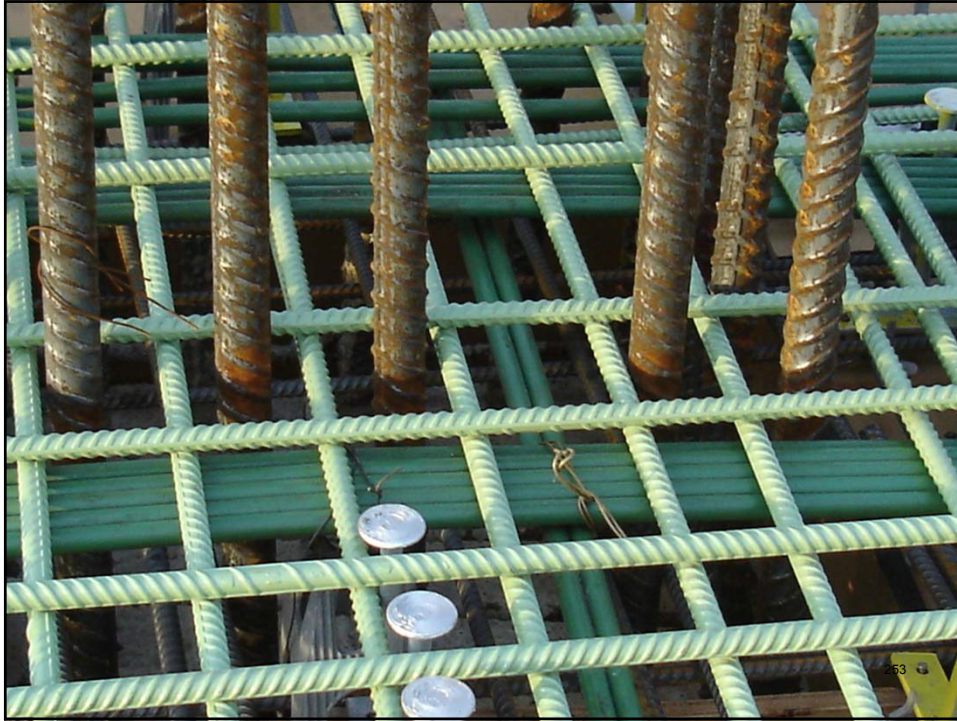
Check Layering of PT and Mild Steel Reinforcement

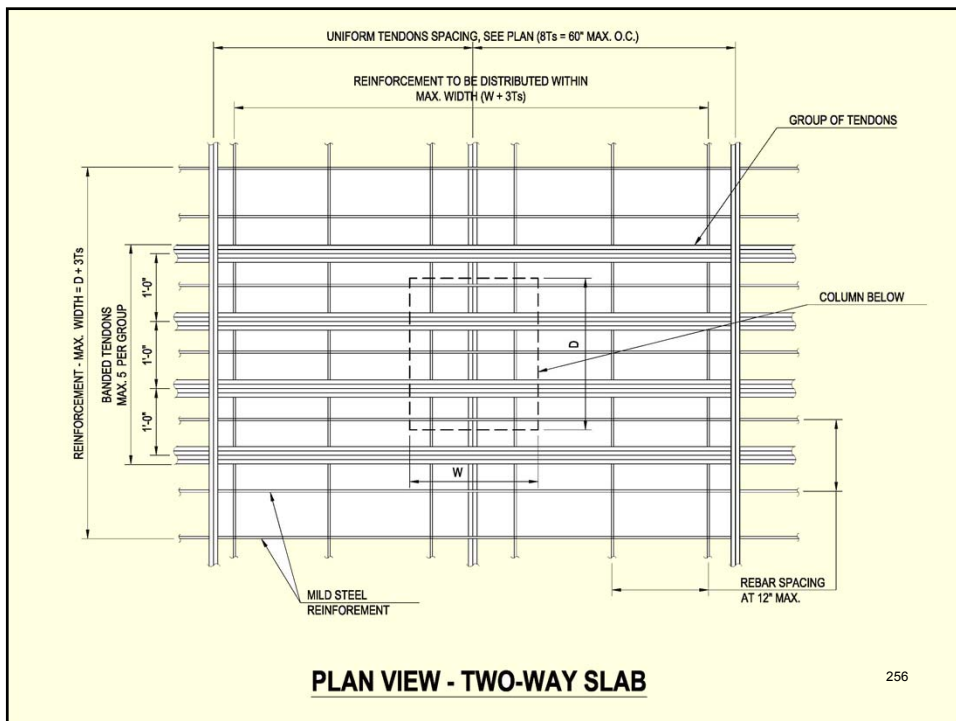
- This should follow the layout diagram on the drawings. This layout should give the sequencing of placement of mild steel and tendons. The top load/mat gives the most problems. Some weaving may be involved.
- Ideally to get the maximum benefit (max. drape) of prestress – 2 layers should be used
- Sometimes for convenience 3 layers are acceptable
- The easiest is placing in 4 layers. This means that the tendons in both directions are placed then the rebar mats go on top in the two directions.

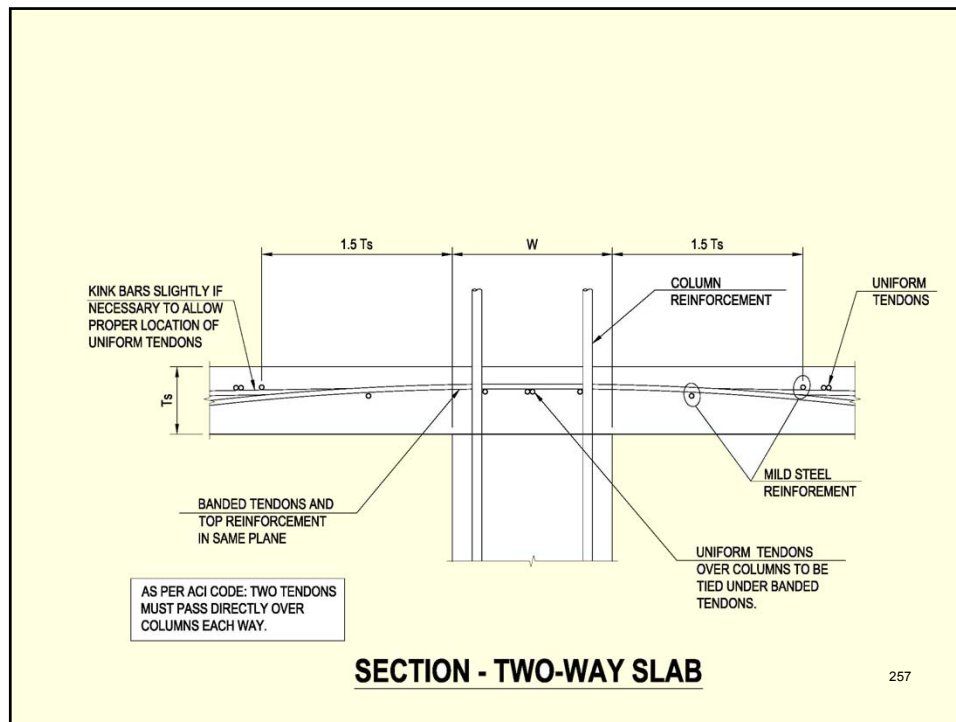
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FIELD ISSUES

PRIOR TO CONCRETE PLACEMENT – contd.

Review Sleeves/ Holes in Anchorage Zones

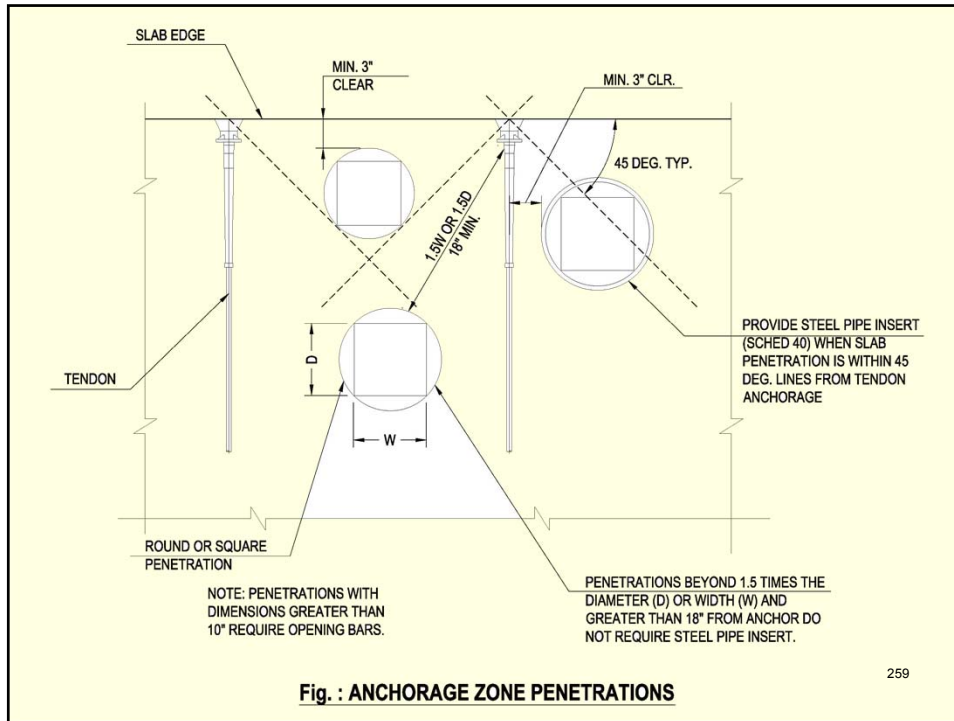
- Area behind anchors – high stress
- Roughly 18" bounded by 45° lines starting at the anchor – No sleeves shall be placed in this zone; If absolutely necessary then steel pipe sleeves shall be used
- See sketch for restrictions

Check if there is adequate stressing length of tendons available beyond the form edge

- Minimum 12" – preferably 14"
- This is required to engage the strand by the hydraulic jack
- If less than 12" special equipment may be needed

Attachment of anchorage device and pocket formers

- Are anchors secured with nails and/or screws to edge form?
If not then concrete slurry can penetrate the pocket and create problem for seating of wedges



FIELD ISSUES

PRIOR TO CONCRETE PLACEMENT – contd.

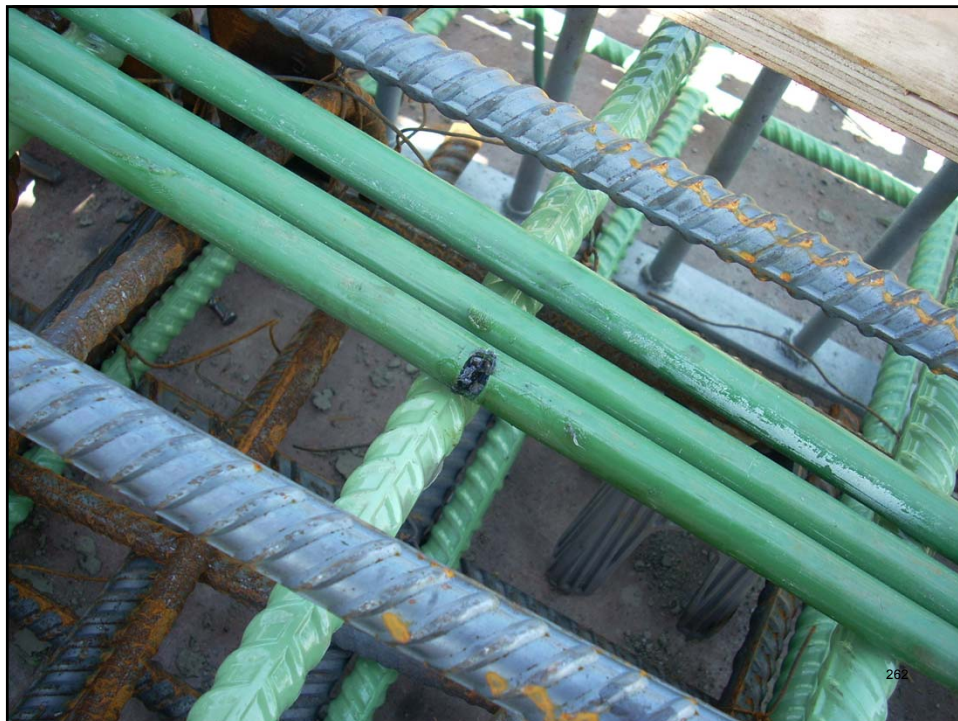
Observe if couplers are adequately installed

- Couplers may be needed if:
- Incorrect tendon fabrication leaving it short – better to replace the tendon with longer tendon if possible. If the preceding tendon is already stressed (intermediate stressing) then couplers may be the only choice
- Tendons are placed in wrong locations then what was detailed
- Accidentally cutting the tail of the tendon

Exposed or damaged strand

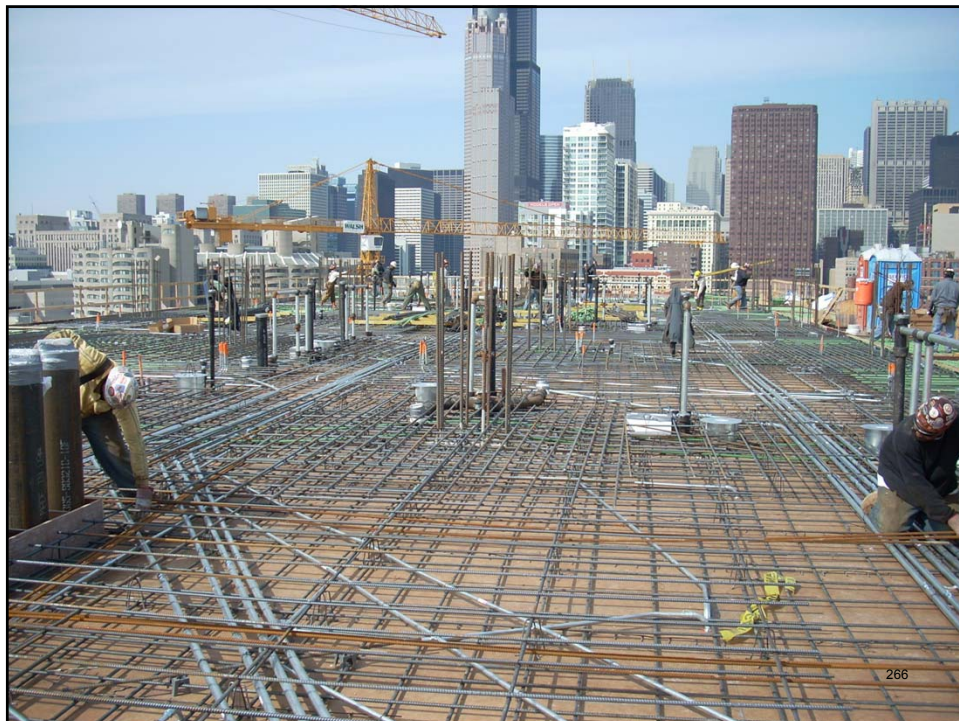
- In non-encapsulated (standard) systems a part of the strand may be exposed at the stressing anchorage which may alter the behavior of the tendon (friction loss) and also reduce the effectiveness of corrosion protection. Can be fixed by wrapping securely with tape
- There may be tears in the sheathing which should be repaired
- Note sheathing thickness = 0.05” = 50 mils

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FIELD ISSUES

PRIOR TO CONCRETE PLACEMENT – contd.

Check horizontal curvature

Tendons may be curved in a horizontal plane to avoid opening or other obstructions.

- If slope exceeds 1:12 then provide hairpin bars; #3@12" o.c min
- If slope exceeds 1:6 then revise layout
- 2" of separation shall be maintained between the tendons

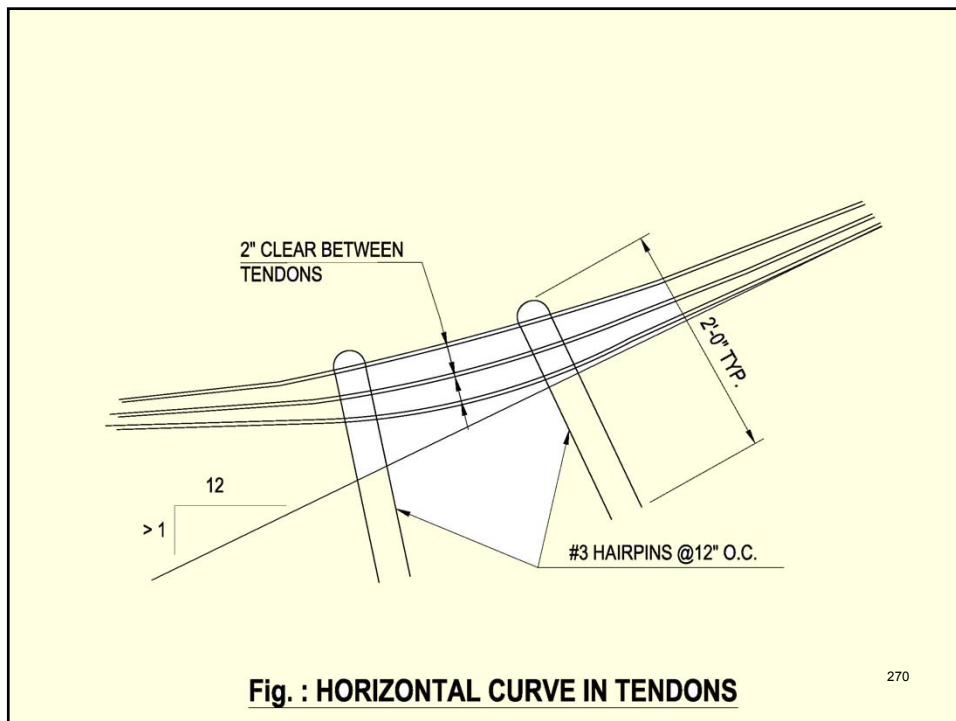
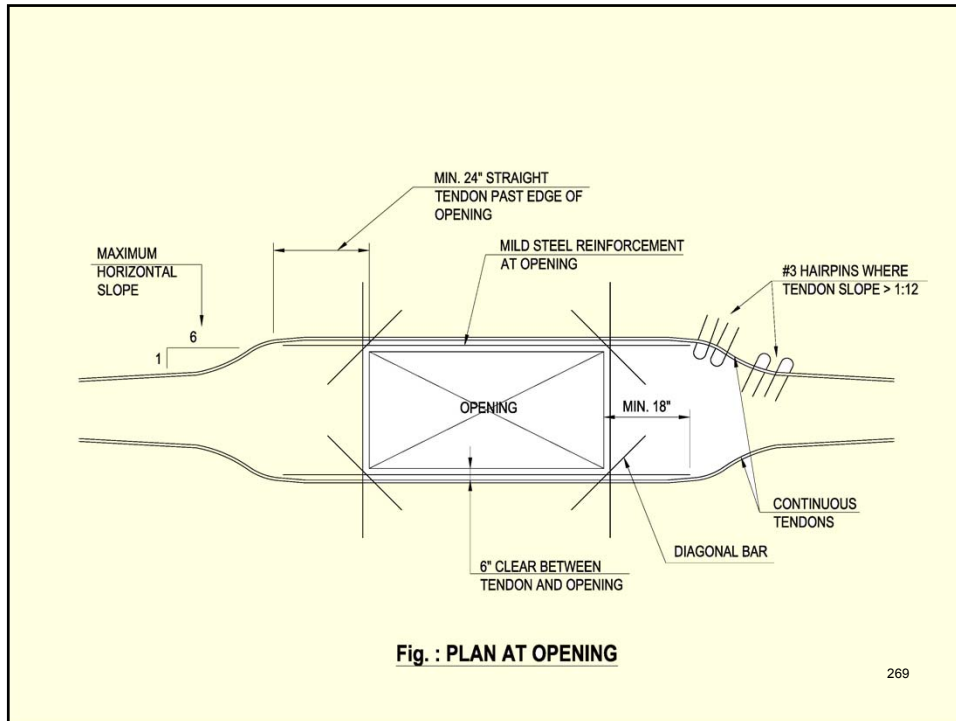
Tendon Adjustment

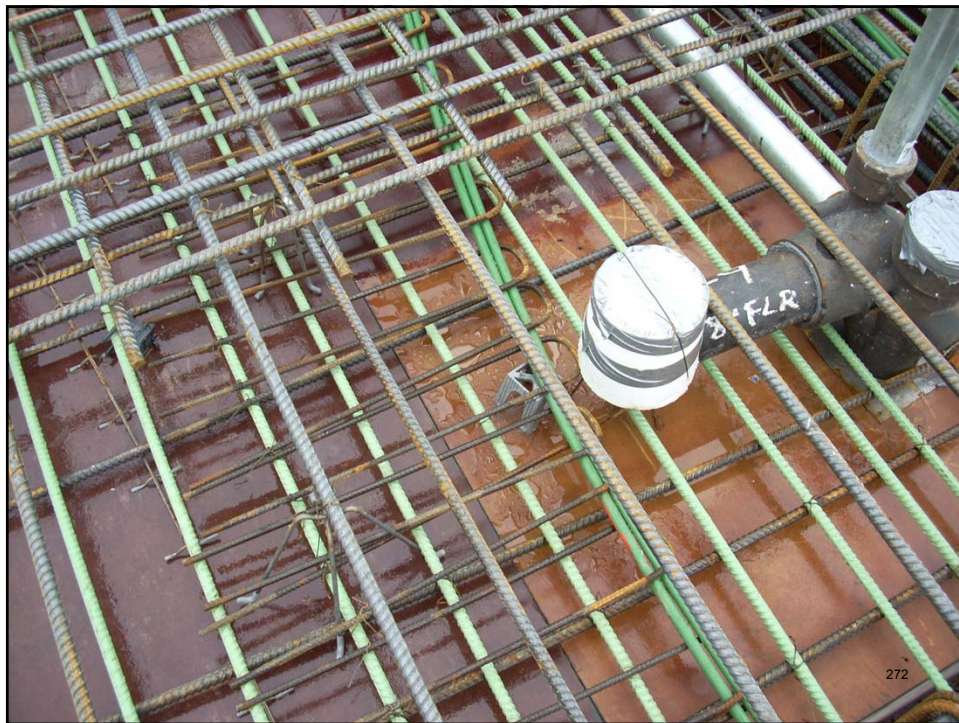
- Tendon locations govern over mild steel.
- Small deviations in horizontal spacing is permitted when required to avoid openings, inserts and dowels.

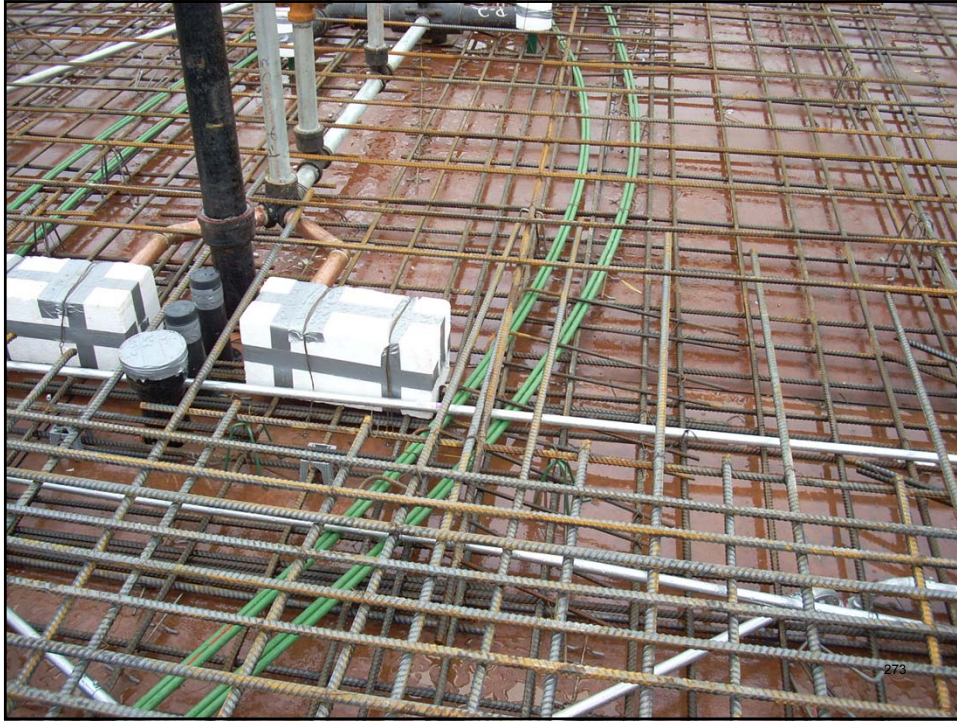
Chairs & Support Bars

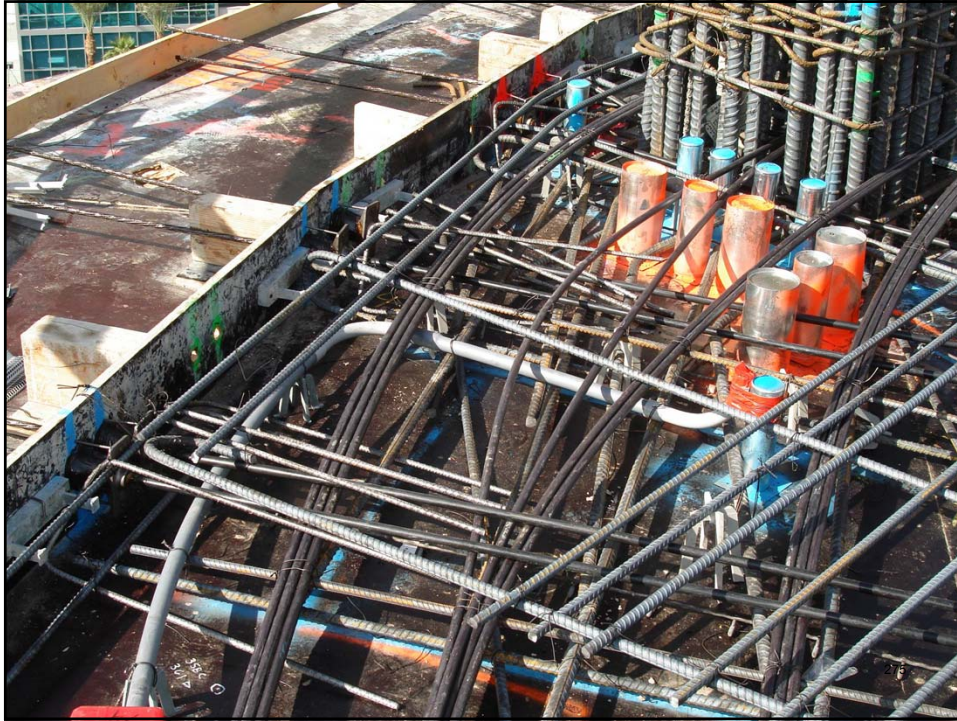
- Chairs shall be spaced at a max of 4' o.c.
- Support bars over chairs shall be min #4 bars

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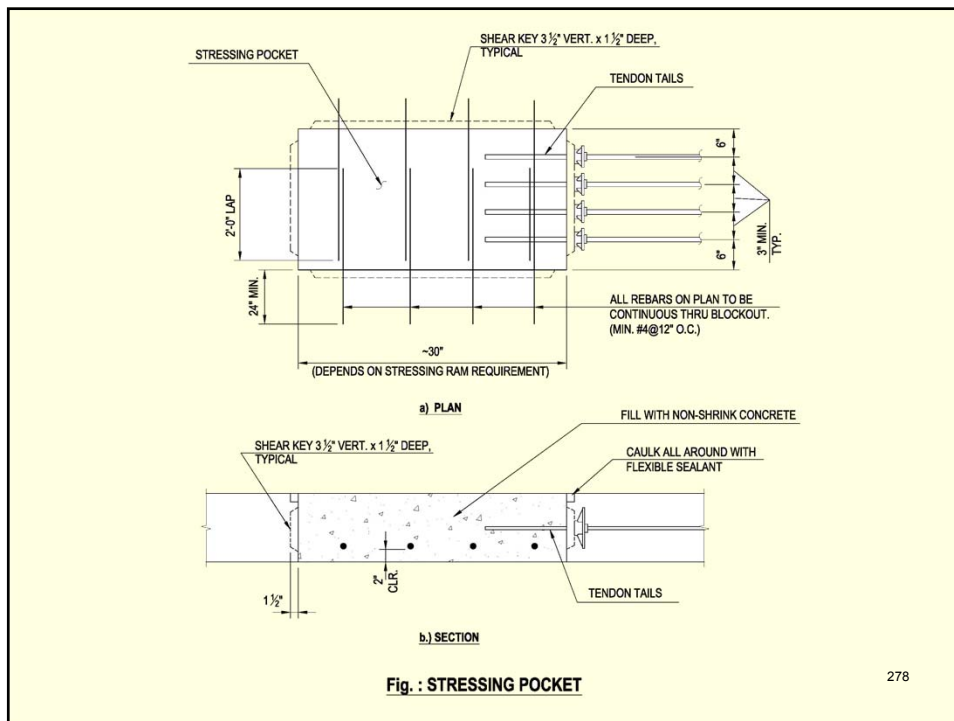




STRESSING POCKETS

- Sometimes these are provided to stress short add tendons – see sketch
- Check if the detailing of reinforcement is properly done at these stressing pockets

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FIELD ISSUES

AFTER CONCRETE PLACEMENT

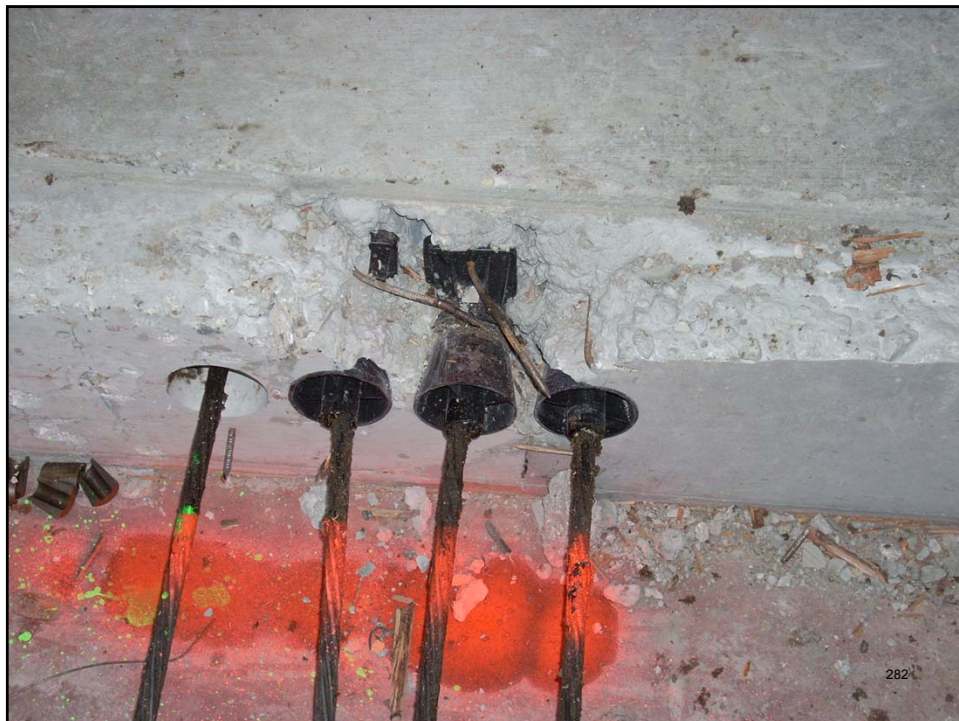
Honeycombing

- All honeycombing and voids in anchorage zone shall be repaired before tendons are stressed.
- Remove all loose materials
- Patch with high-strength, non-shrink concrete repair mix
- Stress only after concrete has reached required strength

Blow-Outs

- Rupture of concrete when tensile stresses exceeds material strength.
- Typically they occur at the anchorages during stressing operation
- Caused by voids in concrete due to rebar congestion at anchorages or under-strength of concrete or cold joint in anchorage zone.
- Quite often chunks of concrete come loose and are thrust into air
- For repair the tendons are detensioned in the immediate vicinity of blow-out. Loose materials are removed and the anchor is reset. If strand is damaged then it is cut-off and new length of strand is coupled to it. Concrete is patched with high strength non-shrink repair mix
- Finally the tendon is re-stressed.
- Blow-outs can also happen at high & low point of tendon profile and at excessive horizontal curvature

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FIELD ISSUES

AFTER CONCRETE PLACEMENT – contd.

Out of range elongations

- The field measured elongations are measured against the calculated elongations.
- This gives an indication whether the tendons are delivering the expected force to the concrete.
- If the elongations are within 5-7% of theoretical then they are deemed acceptable
- If actual elongation > calculated elongation then the jack may be applying too much force or the concrete may be green when stressed.
- If actual elongation < calculated elongation then there may be too much friction along the tendon and the losses are significant. Tendon may be exposed at some point along its length.
- The discrepancy should be investigated and removed.

Lift-Off Procedures

- Purpose is to determine the actual force in the stressed tendon
- This is done if there are unexplainable differences between the calculated & recorded elongations
- Test performed before the tendon tail is cut-off
- The test is done with standard stressing jack
- The pressure shown on gage attached to jack will rise until the wedges break loose
- When this happens there will be an immediate drop in gage pressure.
- This lower pressure is recorded as the actual force in the tendon.
- The tendon is then prestressed to its original force and locked off with wedges.

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FIELD ISSUES

AFTER CONCRETE PLACEMENT – contd.

Future Openings

- The tendon locations can be marked on the forms with chalk or paint before placing concrete. The markings are transferred to the concrete soffit when the concrete is placed leaving a clear record of tendon locations
- For large openings like new stair openings, tendons may be cut in a controlled environment and the floor will be retrofitted with new concrete beams or steel beams. External pre-stressing option is also available.

Anchors into Concrete

- Careful with penetrations – do not damage PT tendon
- The tendons are normally spaced apart in the uniform direction and grouped very closely in the banded direction. As such if the tendon locations are established it will be easy to drive an anchor through the floor system.
- The tendon locations may be marked on the soffit as discussed above or radiographic methods may be used to locate them
- These include X-Rays and GPR (Ground Penetrating Radar)
- Anchors with penetration less than the concrete cover may be used.
- Normally for garage slab the clear cover is 1 ½" to 2" depending on Code and this distance can be used for anchor embedment
- For ceilings drop-in anchors are commonly used. Hilti HD1-P is one such anchor with ¾" embedment ²⁸⁶

FIELD ISSUES

AFTER CONCRETE PLACEMENT – contd.

Strand Breakage

- The strand typically consists of 7 wires. It is possible that one or more wire may break
- This can happen due to overstressing, internal damage to the tendon or misalignment of the wedges.
- If one or more wires have broken and the strand was released it may be possible to re-stress the remaining wires.
- Dummy wires are inserted so that the jack can hold the strand.
- The tendon is then stressed to $x/7$ of its full load; x being the number of intact wires.

Concrete Strength at Stressing

- Minimum concrete strength shall be 3000 psi at time of stressing of mono-strand tendons
- Tendons to be stressed within 72 hrs of concrete placement to avoid plastic shrinkage cracks
- Min concrete strength shall be established by cylinder breaks

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FIELD CONSIDERATION

AFTER CONCRETE PLACEMENT – contd.

Stressing Sequence

For One-Way Slab and Beams

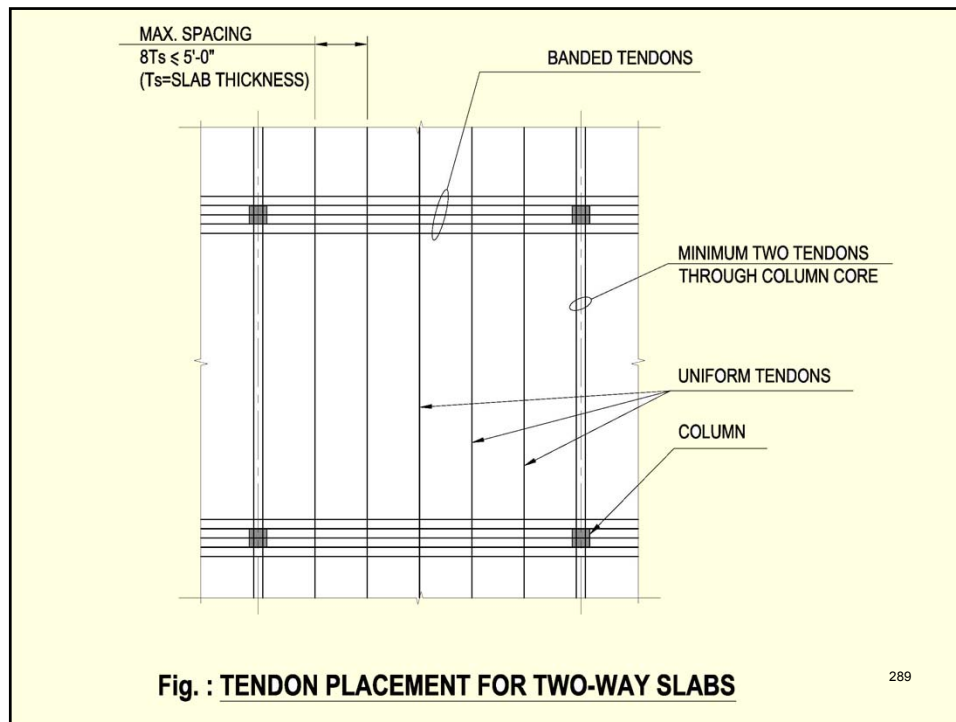
- Stress continuous uniform slab tendons first
- Stress temperature tendons
- Stress beam tendons
- Stress add tendons

For Two-Way Slab

- Stress continuous uniform slab tendons first
- Stress banded tendons
- Stress added distributed tendons
- Stress added tendons in banded direction

In general the uniform tendons are stressed before the banded or beam tendons

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ADDITIONAL POINTS TO NOTE

Core Drilling

- Shall not be allowed without prior approval of the Engineer of Record (EOR)
- Indiscriminate drilling may damage or destroy the tendon
- The EOR may be able to provide guidance as to where core drilling may be acceptable
- This is because the tendons are banded in one direction and spaced apart (usually 3' to 5') in the uniform direction. This gives some leeway.
- Tendon markings on the soffit helps in this regards

Power Driven Fasteners

- Shall not be used unless specific direction is obtained from the EOR.
- At column locations and vicinity, the tendons are in the top part of the slab – power fasteners may be allowed at the bottom of the slab
- At mid-spans and vicinity, the tendons are in the bottom of the slab – power fasteners may be allowed at the top of the slab
- The EOR may create a diagram of a safe area where power driven fasteners may be used
- Power driven fasteners with shallow embedment may be allowed if the cover to PT tendon exceeds the embedment by a comfortable margin. The EOR is the final judge on this

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ADDITIONAL POINTS TO NOTE

Use of Embeds in lieu of Drilled Anchors

- Permanent fixtures supported off the PT slab shall be attached using embeds
- Embed usually a plate with min. of two headed studs. The size of plate and the number and embedment of studs shall be from design unless for very light loads
- Use of drilled anchors is discouraged

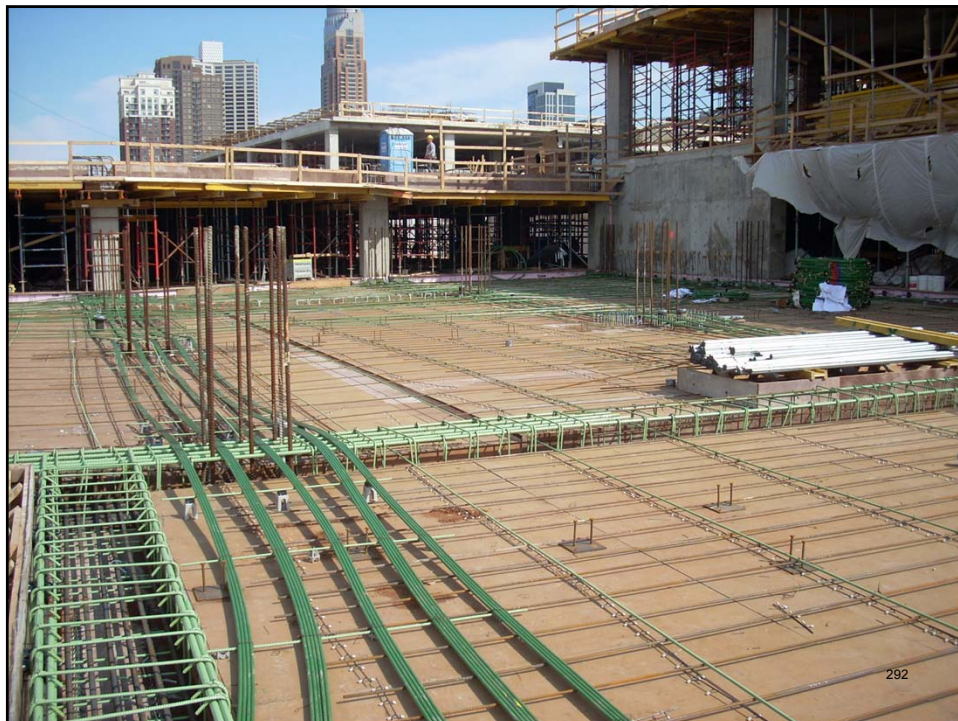
Tendon Grouping

Banded Direction – Max of 5 tendons allowed per group unless approved by EOR
 Uniform Direction – Typically 2 tendons at spacing subject to min of 8T or 5'-0" o.c.

Tendon Tails

- These shall not be cut until all tendons are stressed, elongations measured and the EOR's approval is obtained
- For encapsulated systems the tendon ends shall be protected with a grease filled watertight cap.

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ADDITIONAL POINTS TO NOTE

Shoring

- Shoring may be removed when all tendons are satisfactorily stressed and the EOR's approval is obtained
- Exception – Staged prestressing such as in transfer girder
- In partial spans such as pour strips the shoring in the partial plan shall stay in place until the remaining section of span is poured and stressed.
- The immediate back span may also need to be shored until the span with the pour strip is completed.
- The shoring guidelines at the pour strip shall be provided by the engineer on the drawings.

Tendon Stressing at Previously Cast Wall

- See attached details for various options available

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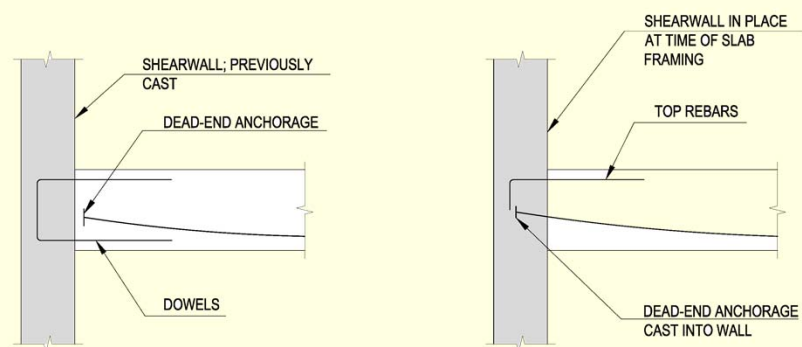
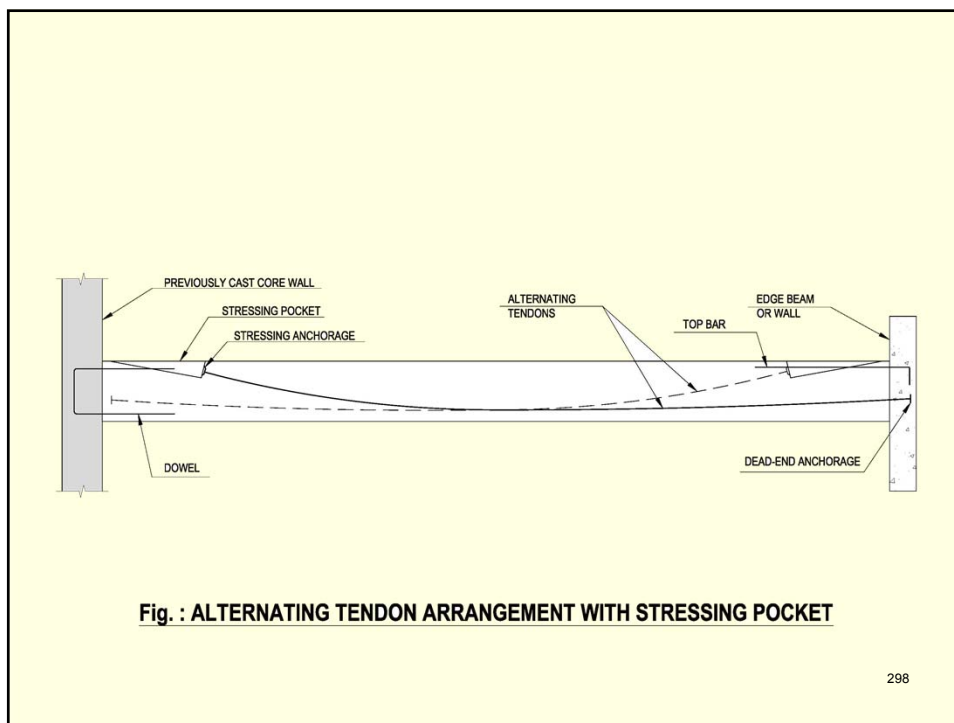
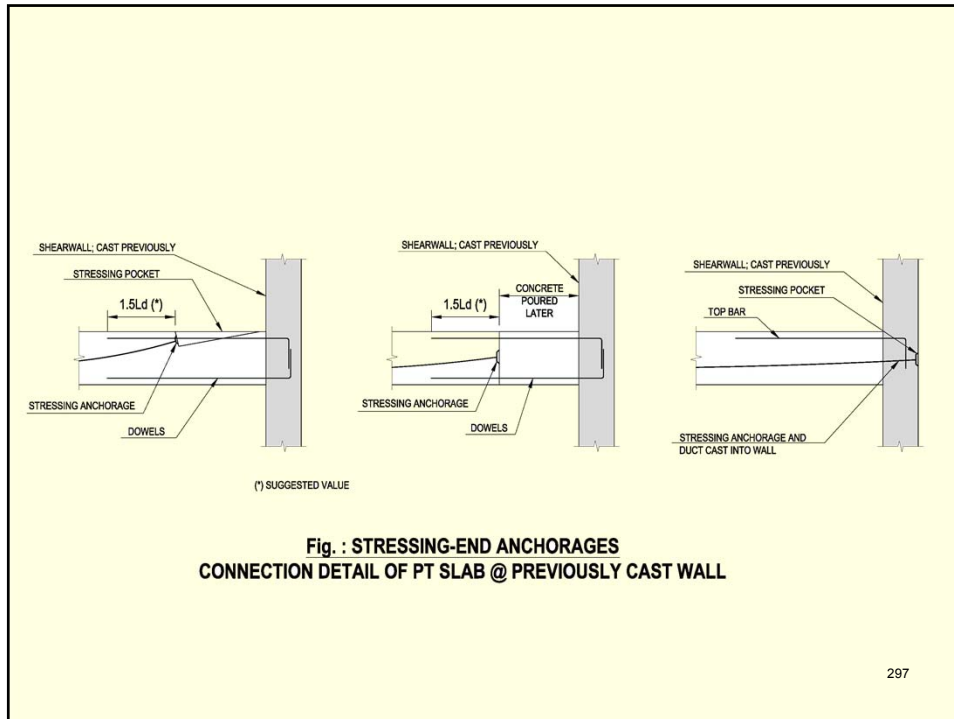
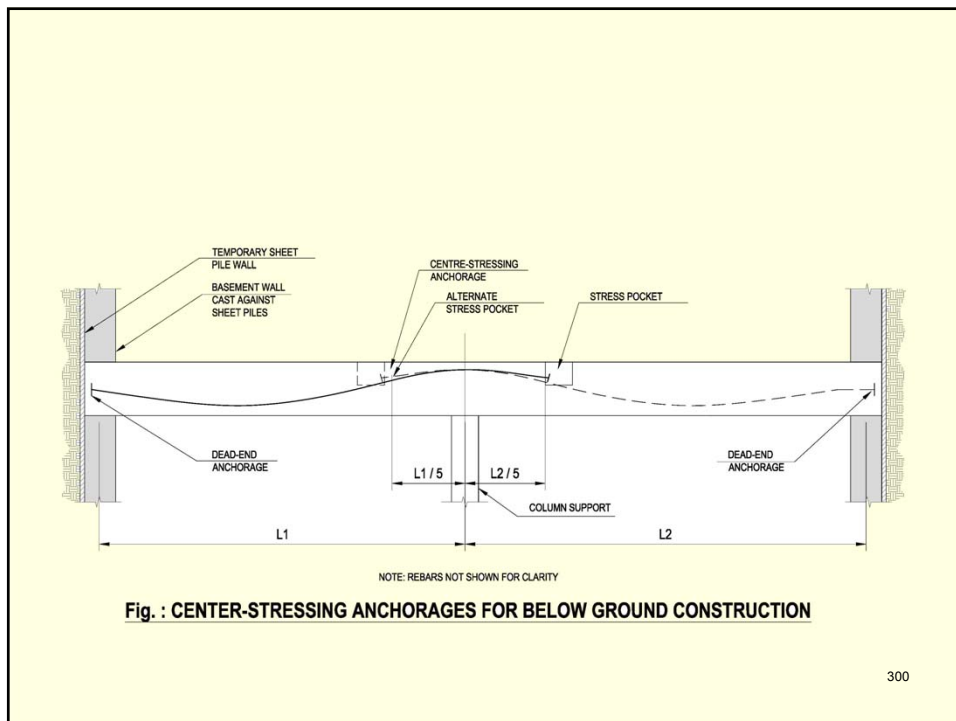
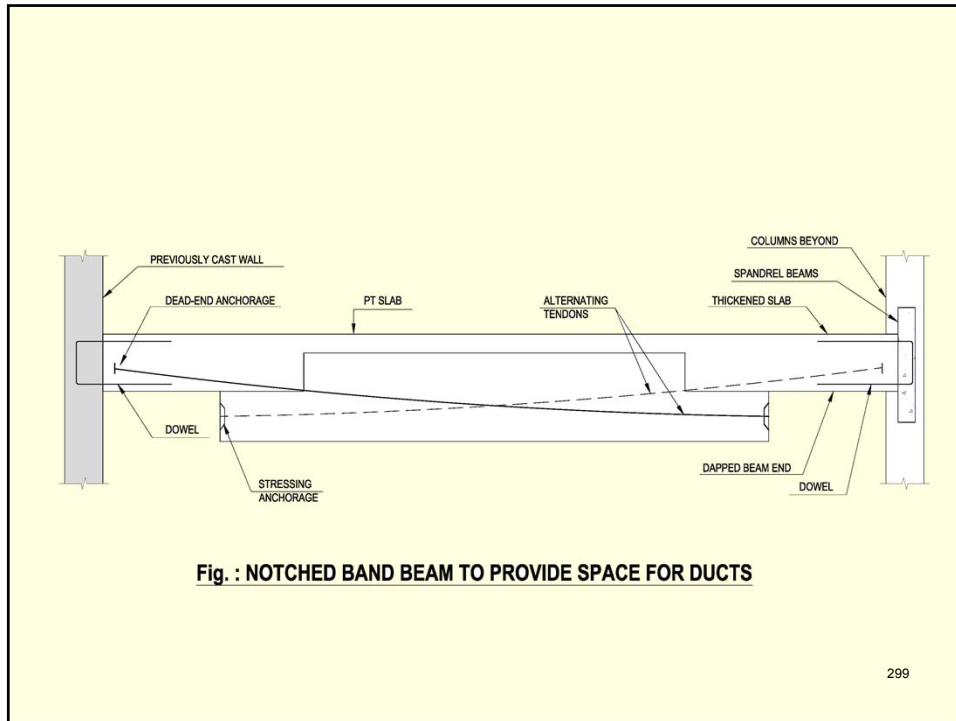


Fig. : DEAD-END ANCHORAGES
CONNECTION DETAIL OF PT SLAB @ PREVIOUSLY CAST WALL ²⁹⁶





RECAP OF EXAMPLES IN SEMINAR NOTES

- Example 1: Computation of Extreme Fiber Stresses by Basic Method
- Example 2: Extreme Fiber Stresses by Load Balancing Method
- Example 3: Average PT force furnished by 1/2" dia strand after losses.
- Example 4: Flexural strength of PT member with bonded and unbonded tendons
- Example 5: Computation of Cracking moment strength with bonded reinf.
- Example 6: Design of PT flat plate using the Equivalent Frame method.
- Example 7: Shear Design of PT Member
- Example 8: Barrier Cable Design
- Example 9: Anchorage Zone Design – 1
- Example 10: Anchorage Zone Design – 2

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HISTORY OF PRESTRESSING

Eugene Freyssinet, France 1928 – Invented Prestressing using High Strength wires

- Prior attempts were made with lower strength steel and the prestress was lost due to losses
- Higher strength steel elongates 0.7% and after accounting for 0.1% creep & shrinkage in concrete still retained 80% of prestress
- Eugene designed conical wedges and special jacks

Gustave Magnel, Belgium 1940 – Used curved multi-wire tendons in flexible rect. ducts

- Magnel system which still being used.
- UK, Germany, Netherland & Italy followed with slightly different systems

US Applications

- First Project – Walnut Lane Bridge, Philadelphia, 1949; Precast pre-stressed girders
- Building Construction – Mid to late 1950's
- PT then became popular in Lift Slab Construction Method
- Unbonded tendons in Building Construction – 1960's
- Use of PT increased 5 fold from 1965-1985
- 8.5% annual growth since 1985
- Button headed tendon system was used early one but had problems
- First strand tendon anchorage developed by Ed Rice of Atlas Prestress Corporation – 1/2" dia 7 wire strand and a plate anchored with 2 – 1/2" wedge chucks
- In 1970's single strand tendon with ductile iron casting which replaced button headed tendons

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ANY QUESTIONS?

Thank you for your participation!

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